

Hyperbolic Functions – Q13 [Practice/E](17/6/23)

(i) Use $\operatorname{artanh}x = \frac{1}{2} \ln \left(\frac{1+x}{1-x} \right)$ to show that $\frac{d}{dx} \operatorname{artanh}x = \frac{1}{1-x^2}$

(ii) Use $\operatorname{arcoth}x = \frac{1}{2} \ln \left(\frac{1+x}{x-1} \right)$ to show that $\frac{d}{dx} \operatorname{arcoth}x = \frac{1}{1-x^2}$ also

Solution

$$\begin{aligned} \text{(i)} \quad \frac{d}{dx} \operatorname{artanh} x &= \frac{1}{2} \cdot \frac{1-x}{1+x} \cdot \frac{(1-x)-(1+x)(-1)}{(1-x)^2} \\ &= \frac{1}{2} \cdot \frac{2}{(1+x)(1-x)} = \frac{1}{1-x^2} \end{aligned}$$

$$\begin{aligned} \text{(ii)} \quad \frac{d}{dx} \operatorname{arcoth} x &= \frac{1}{2} \cdot \frac{x-1}{1+x} \cdot \frac{(x-1)-(1+x)}{(x-1)^2} \\ &= \frac{1}{2} \cdot \frac{-2}{(1+x)(x-1)} = \frac{1}{1-x^2} \end{aligned}$$