

Hyperbolic Functions – Q11 [Problem/M](17/6/23)

What is the domain of $\operatorname{artanh}\left(\frac{x}{2}\right)$?

Solution

$$\tanh x = \frac{\sinh x}{\cosh x} = \frac{e^x - e^{-x}}{e^x + e^{-x}} = \frac{e^{2x} - 1}{e^{2x} + 1} = \frac{e^{2x} + 1}{e^{2x} + 1} - \frac{2}{e^{2x} + 1} = 1 - \frac{2}{e^{2x} + 1}$$

Thus $-1 < \tanh x < 1$ (as $x \rightarrow -\infty$ & ∞)

As $\operatorname{artanh} x$ is the inverse of $\tanh x$, the domain of $\operatorname{artanh} x$ is the range of $\tanh x$; ie $(-1, 1)$.

Thus the domain of $\operatorname{artanh}\left(\frac{x}{2}\right)$ satisfies $-1 < \frac{x}{2} < 1$;

ie $-2 < x < 2$