Hyperbolic Functions Overview (17/6/23)

Q1 [Practice/E]

(i) Prove, using exponential functions, that

(a) $cosh^2x - sinh^2x = 1$

(b) $sinh2x = 2sinhxcoshx$

(ii) By differentiating the result from (i)(b), obtain an expression for $cosh2x$ in terms of $cosh^2x$ and $sinh^2x$

Q2 [Practice/E]

(a) Find the formula connecting $tanh^2x$ & $sech^2x$?

(b) Find the formula connecting $coth^2x$ & co$sech^2x$?

Q3 [Practice/E]

If $x = sinhu$, write $sinh(4u)$ in terms of x

Q4 [Practice/M]

Given that $\int \frac{1}{\sqrt{x^2-a^2}}dx = arcosh(\frac{x}{a})$, and that

$arcoshx = ln\,(x + \sqrt{x^2 - 1})$, justify the writing of the integral as ln $(x + \sqrt{x^2 - a^2})$

Q5 [Practice/E]

Find or prove the following:

(i) $\frac{d}{dx}\ tanhx$

(ii) $\frac{d}{dx} arcoshx = \ \frac{1}{\sqrt{x^2-1}}$

(iii) $\frac{d}{dx}\ artanhx = \ \frac{1}{1-x^2}$

(iv) $\frac{d}{dx}\ sechx$

Q6 [Practice/M]

Solve the equation $5cosh2x + 3sinhx = 6$,

giving your answers in exact logarithmic form.

Q7 [Practice/E]

Using the logarithmic form of $arcoshx$, prove that the derivative of $arcoshx$ is $\frac{1}{\sqrt{x^2-1}}$

Q8 [Practice/M]

Show that $artanhx = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right) \quad (|x| < 1)$

Q9 [Practice/H]

Show that $arcoshx = \ln\left(x + \sqrt{x^2 - 1}\right)$

Q10 [Practice/M]

Derive an expression for $arsinh(a)$ in the form lnb

Q11 [Problem/M]

What is the domain of $artanh\left(\frac{x}{2}\right)$?

Q12 [Practice/M]

Show that $arcothx = \frac{1}{2}ln\left(\frac{1+x}{x-1}\right) \quad (|x| > 1)$

Q13 [Practice/E]

(i) Use $artanhx = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right)$ to show that $\frac{d}{dx}artanhx = \frac{1}{1-x^2}$

(ii) Use $arcothx = \frac{1}{2}\ln\left(\frac{1+x}{x-1}\right)$ to show that $\frac{d}{dx}arcothx = \frac{1}{1-x^2}$ also

Q14 [Problem/M]

(i) Show that $arcothx = artanh\left(\frac{1}{x}\right)$

(ii) Find $f(x)$ such that $arcoshx = arsinh(f(x))$

Q15 [Practice/M]

Given that $arcoshx = ln(x + \sqrt{x^2 - 1})$, show that if

$cosha = b$ then $a = \ln\left(b \pm \sqrt{b^2 - 1}\right)$

Q16 [Practice/E]

Write lna in the form $arsinh\left(f(a)\right)$, where $f(a)$ is some expression in terms of a.