Groups – Q2 [18 marks](25/5/21)

Exam Boards

OCR : Add. Pure (Year 1)

MEI: -

AQA: Discrete (Year 2)

Edx: -

(i) Show that the set {1,4,7,13} forms a group, under multiplication modulo 15. [7 marks]

(ii) Find the generators of the group. [6 marks]

(iii) Establish whether the group is cyclic. [2 marks]

(iv) Identify all the subgroups. [3 marks]

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Solution

(i) The Cayley table is:

	1	4	7	13
1	1	4	7	13
4	4	1	13	7
7	7	13	4	1
13	13	7	1	4

[3 marks]

The operation is closed. [1 mark]

There is an identity element (1). [1 mark]

The identity element appears in each row, so that each element has an inverse. [1 mark]

Associativity follows from the associativity of ordinary multiplication. [1 mark]

So the conditions for a group are satisfied.

(ii) $4^2 \equiv 1$ [1 mark]

$7^2 \equiv 4; 7^3 = 4 \times 7 = 28 \equiv 13; 7^4 = 13 \times 7 = 91 \equiv 1$ [2 marks]

$13^2 \equiv 4; 13^3 = 4 \times 13 = 52 \equiv 7; 13^4 = 7 \times 13 = 91 \equiv 1$

[2 marks]

So 7 and 13 are the generators of the group. [1 mark]

(iii) The elements of the group can be written as:

$1, 7, 7^2, 7^3$ (for example), and so the group is cyclic. [2 marks]

[Also, the elements have periods 1,2,4,4, which is the pattern for a cyclic group of order 4.]

(iv) The subgroups are $\{1\}, \{1,4\}, \{1,7,4,13\}$ [3 marks]