Groups – Q1 [8 marks](25/5/21)

Exam Boards

OCR : Add. Pure (Year 1)

MEI: -

AQA: Discrete (Year 1)

Edx: -

Multiplication modulo m (or just mod m), denoted by $\times_m$, is defined on the set $\{0,1,2,\dots,m-1\}$ by carrying out ordinary multiplication and taking the remainder when the product is divided by m. For example, $5 \times_3 4 = 2$.

Show that $\times_5$ is a closed and commutative binary operation on the set $\{0,1,2,\dots,4\}$, and identify the inverse of each element, where it exists. [8 marks]

Multiplication modulo m (or just mod m), denoted by $\times_m$, is defined on the set $\{0,1,2, \dots, m-1\}$ by carrying out ordinary multiplication and taking the remainder when the product is divided by m. For example, $5 \times_3 4 = 2$.

Show that $\times_5$ is a closed and commutative binary operation on the set $\{0,1,2, \dots, 4\}$, and identify the inverse of each element, where it exists. [8 marks]

Solution

(i) The Cayley table is:

$\times_5$	0	1	2	3	4
0	0	0	0	0	0
1	0	1	2	3	4
2	0	2	4	1	3
3	0	3	1	4	2
4	0	4	3	2	1

[3 marks]

As $\times_5$ is defined for all pairs of elements of the set, it is a binary operation. [1 mark]

Each of the products in the Cayley table is an element of the set, and so the operation is closed. [1 mark]

It is commuative because of symmetry about the leading diagonal. [1 mark]

[Note: Associativity is harder to establish, but can usually just be asserted in the case of modular addition or multiplication.]

The element 1 leaves all elements unchanged, and is therefore the identity element. [1 mark]

The inverses are as follows:
0: doesn't exist
1: 1
2: 3
3: 2
4: 4

[1 mark]