Graphs – Q2 [Practice/M]

Prove that the number of arcs in K_n is $\frac{1}{2}(n-1)n$?

Solution

Method 1

There are n nodes of order $n - 1$

So total of orders $= n(n - 1)$,

and total number of arcs $= \frac{1}{2}n(n - 1)$

Method 2

There are $n - 1$ arcs joining the 1st node.

After the 1st node has been excluded, there are a further $n - 2$ arcs joining the 2nd node; and so on.

Giving a total of $(n - 1) + (n - 2) + \cdots + 1 = \frac{1}{2}(n - 1)n$