Game Theory – Q3 [22 marks](28/5/21)

Exam Boards

OCR : D (Year 1)

MEI: -

AQA: D (Year 1)

Edx: D2 (Year 2)

A zero-sum game is given by the following pay-off matrix (from player 1's point of view).

Player 2: Player 1	A	B	C
A	1	-2	2
B	3	4	-1

(i) Confirm that there is no stable solution, and find the optimal mixed strategy for player 1, and their expected pay-off.
[12 marks]

(ii) By using the fact that the expected pay-off for player 2 will equal $-1 \times$ the expected pay-off for player 1, find the optimal mixed strategy for player 2. [10 marks]

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Solution

(i)

Player 2: Player 1	A	B	C	row min.
A	1	-2	2	-2
B	3	4	-1	(-1)
col. max.	3	4	(2)	

[2 marks]

As the max. of the row minima doesn't equal the min. of the col. maxima, there is no stable solution. [1 mark]

To find the strategy for player 1:

Let p be the probability that player 1 chooses option A.

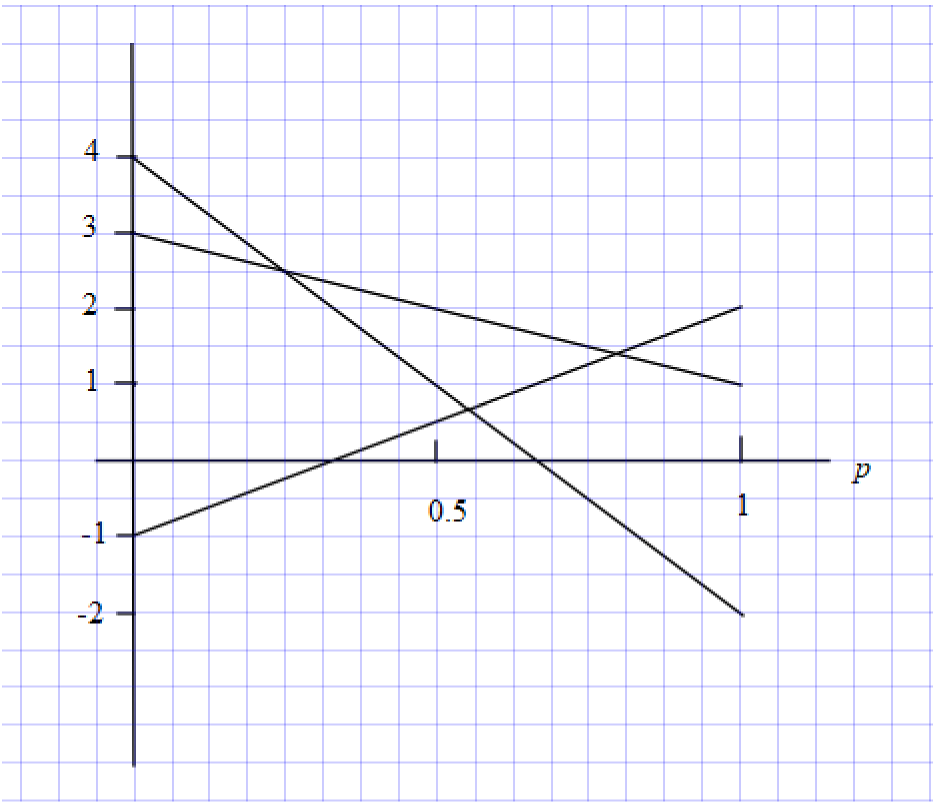
Then the expected pay-off for player 1 if player 2 chooses A is:

$p + 3(1 - p) = 3 - 2p$ [1 mark]

If player 2 chooses B it is: $-2p + 4(1 - p) = 4 - 6p$ [1 mark]

And if player 2 chooses C it is: $2p - (1 - p) = 3p - 1$ [1 mark]

The diagram below shows the lines $y = 3 - 2p$, $y = 4 - 6p$ and $y = 3p - 1$



[2 marks]

The optimal value of p occurs when $min(3 - 2p , 4 - 6p, 3p - 1)$ is maximised, and this occurs at the intersection of the lines

$y = 4 - 6p$ and $y = 3p - 1$ [2 marks]

ie when $4 - 6p = 3p - 1 \Rightarrow 5 = 9p; p = \frac{5}{9}$ [1 mark]

and the expected pay-off for player 1 is $4 - 6\left(\frac{5}{9}\right) = \frac{2}{3}$ [1 mark]

(ii) To find the strategy for player 2:

Let q be the probability that player 2 chooses option A, and r the probability that they choose option B; so that they choose option C with probability

$1 - q - r$.

Then the expected pay-off for player 2 if player 1 chooses A is:

$(-1)q + 2r - 2(1 - q - r) = q - 2 + 4r$ [2 marks]

If player 1 chooses B it is: $-3q - 4r + (1 - q - r) = -4q - 5r + 1$ [1 mark]

The probability rule is chosen in such a way that the expected pay-offs are the same , whichever option player 1 chooses, and both are equal to the value of the game from player 2's point of view, which is known to be $-\frac{2}{3}$. [2 marks]

So $q + 4r - 2 = -\frac{2}{3}$ and $-4q - 5r + 1 = -\frac{2}{3}$ [2 marks]

$\Rightarrow 3q + 12r - 6 = -2$ or $3q + 12r = 4$ (1)

and $-12q - 15r + 3 = -2$ or $12q + 15r = 5$ (2)

Then $4 \times (1) - (2) \Rightarrow 33r = 11; r = \frac{1}{3} \Rightarrow q = 0$ [2 marks]

So the probability rule for player 2 is:

Choose A with probability 0

Choose B with probability $\frac{1}{3}$

Choose C with probability $\frac{2}{3}$ [1 mark]