

## Game Theory Overview (28/5/21)

### Q1 [14 marks]

(i) The following pay-off matrix is for a zero-sum game (from player 1's point of view).

| Player 2: | A  | B | C  | D  |
|-----------|----|---|----|----|
| Player 1  |    |   |    |    |
| A         | 4  | 3 | 2  | 0  |
| B         | 3  | 3 | -1 | -2 |
| C         | -2 | 2 | 3  | 1  |

Use the idea of dominance to reduce the matrix as much as possible. [4 marks]

(ii) Identify the play-safe strategies for players 1 and 2. Explain whether or not there is a stable solution. [5 marks]

(iii) What will be the outcome of the game if:

(a) both players play safe

(b) player 1 plays safe, and player 2 hears of player 1's intention

(c) player 2 plays safe, and player 1 hears of player 2's intention

[5 marks]

### Q2 [12 marks]

A zero-sum game is given by the following pay-off matrix (from player 1's point of view). Confirm that there is no stable solution, and find the optimal mixed strategy for each player, and their expected pay-offs.

|           |   |    |
|-----------|---|----|
| Player 2: | A | B  |
| Player 1  |   |    |
| A         | 2 | 3  |
| B         | 4 | -1 |

### Q3 [22 marks]

A zero-sum game is given by the following pay-off matrix (from player 1's point of view).

|           |   |    |    |
|-----------|---|----|----|
| Player 2: | A | B  | C  |
| Player 1  |   |    |    |
| A         | 1 | -2 | 2  |
| B         | 3 | 4  | -1 |

(i) Confirm that there is no stable solution, and find the optimal mixed strategy for player 1, and their expected pay-off.

[12 marks]

(ii) By using the fact that the expected pay-off for player 2 will equal  $-1 \times$  the expected pay-off for player 1, find the optimal mixed strategy for player 2. [10 marks]