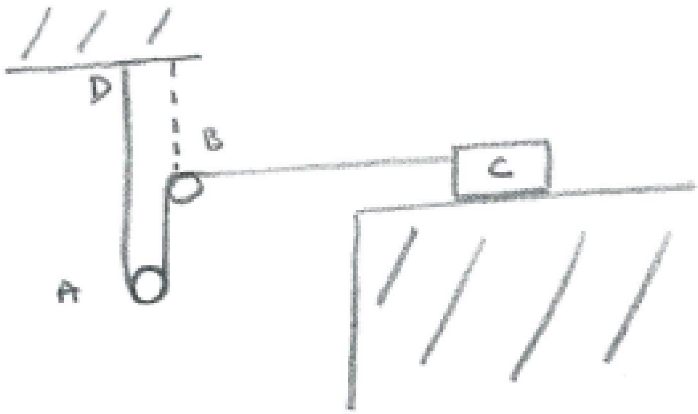
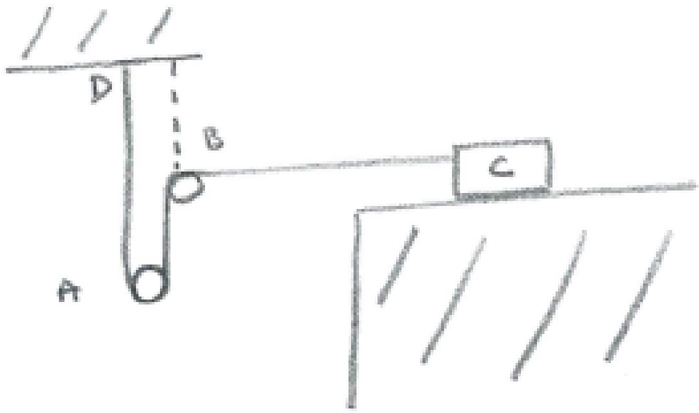
Friction – Q4 [Problem/H] (3/6/21)



Referring to the diagram, A is a smooth pulley of mass 2 kg, which can move up and down; B is a smooth, fixed pulley, and C is a block of mass 1kg, which is initially held at rest on a table. A light inextensible rope is fixed at D, and leads to C, via the two pulleys.

C is now released and accelerates at 2 ms^{-1}. Find the coefficient of friction, μ between C and the table.



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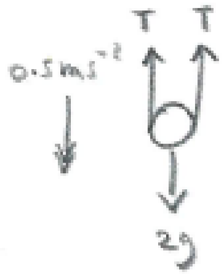
Solution

First of all, the tension (T, say) is the same throughout the rope (since the rope is light and the pulleys are smooth - see note (i) below).

Also, because A falls by half the distance that C moves, its acceleration is also half that of C - see note (ii) below).

[The fact that the rope is inextensible ensures that the rope (as a whole) and the block move the same distances.]



From the force diagram for A,

N2L ⇒ $2g - 2T = (2)(1) \Rightarrow T = g - 1$ [1]

From the force diagram for B,

N2L ⇒ $T - \mu R = (1)(2)$

Also, vertical equilibrium ⇒ $R = g$,

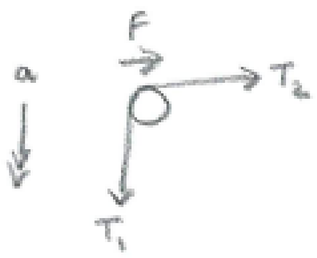
so that $T - \mu g = 2$ [2]

Then [1]&[2] ⇒ $\mu g = g - 3$,

so that $\mu = 1 - \frac{3}{9.8} = 0.694$ (3sf)

Notes

(i) Referring to the force diagram for the rope around B, for example:



Suppose that the rope has mass m, that the pulley exerts a frictional force F, and that the rope experiences forces T_1 and T_2 at its ends.

If the rope has acceleration a,

then $T_1 - T_2 - F = ma$

If $F = 0$ (ie if the pulley is smooth), and $m \approx 0$ (ie the rope is 'light', and so has negligible mass), then $T_1 - T_2 \approx 0$, and so the tensions are approximately equal.

(ii) From the suvat equation, if $u = 0$, then $s = \frac{1}{2}at^2$, so that the acceleration is proportional to the distance, for a given t.