Friction – Q2 [Problem/M] (3/6/21)

A block rests on a slope which is angled at $\theta°$ to the horizontal. The coefficient of friction between the surface of the slope and the block is $\tan \alpha$. P_1 is the horizontal force that needs to be applied to the block to stop it from slipping down the slope, whilst P_2 is the greatest horizontal force that can be applied without the block slipping up the slope.

(i) Show that $\dfrac{P_2}{P_1} = \dfrac{\tan(\theta+\alpha)}{\tan(\theta-\alpha)}$

(ii) Explain what happens when $\theta < \alpha$

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Solution

(i) As friction acts to oppose attempted motion, the frictional force acts up the slope in the first case, and down in the second.
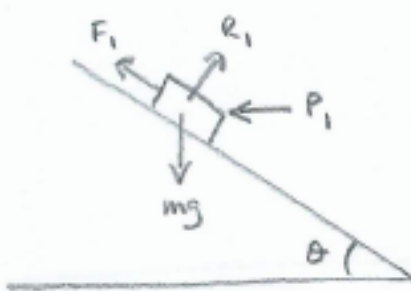
In the first case, resolving forces perpendicular to the slope,
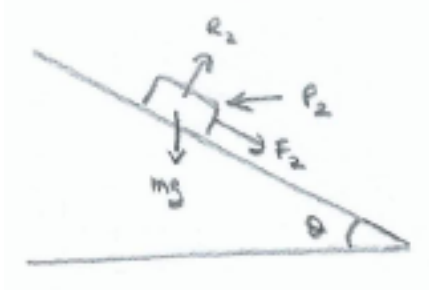
$N2L \Rightarrow R_1 = mg\cos\theta + P_1\sin\theta$

Limiting friction $\Rightarrow F_1 = \tan\alpha R_1$

Resolving forces along the slope,

$N2L \Rightarrow F_1 + P_1\cos\theta = mg\sin\theta$

Hence $\tan\alpha(mg\cos\theta + P_1\sin\theta) + P_1\cos\theta = mg\sin\theta$

and so $P_1(\cos\theta + \tan\alpha\sin\theta) = mg(\sin\theta - \tan\alpha\cos\theta)$ (1)

In the second case, $R_2 = mgcos\theta + P_2sin\theta$, $F_2 = tan\alpha R_2$

and $F_2 + mgsin\theta = P_2cos\theta$

Hence $tan\alpha(mgcos\theta + P_2sin\theta) + mgsin\theta = P_2cos\theta$

and so $P_2(cos\theta - tan\alpha sin\theta) = mg(sin\theta + tan\alpha cos\theta)$ (2)

Then (1) & (2) $\Rightarrow \dfrac{P_2}{P_1} = \dfrac{(sin\theta + tan\alpha cos\theta)(cos\theta + tan\alpha sin\theta)}{(cos\theta - tan\alpha sin\theta)(sin\theta - tan\alpha cos\theta)}$

$= \dfrac{(tan\theta + tan\alpha)(1 + tan\alpha tan\theta)}{(1 - tan\alpha tan\theta)(tan\theta - tan\alpha)} = \dfrac{\tan(\theta + \alpha)}{\tan(\theta - \alpha)}$

(ii) $tan\theta$ is the minimum value of μ required for the block to rest on the slope without slipping. Hence, when $\theta < \alpha$, so that $tan\theta < tan\alpha = \mu$, no horizontal force is needed.