Forces – Q8 [Problem/H] (2/6/21)

A stepladder is made up of two sides, which have weights 80N and 8N. Both sides are of length 2m. There is a platform resting on the top, which together with a person standing on it weighs 700N. The two sides are also joined together by a horizontal light rope of length 1m, which starts at a distance of 0.6m along each side, from the base. See Fig. 1. There is no friction between the ladder and the ground, or between the platform and the ladder. Find the tension in the rope.

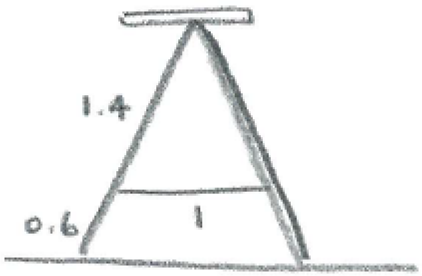

Fig. 1

Solution

[Equations can be obtained by applying N2L and/or taking moments for either individual components (eg one side of the stepladder), or the whole system. There will be a limit to the number of independent equations that can be created. But because some equations may prove to be redundant anyway (as we will see), it may not be worth attempting to ensure that all the equations are independent. Instead, we can just create any equations that look useful, and run the risk of some duplication. Not all of the equations created below are needed for the purpose of finding T, but may be of use for establishing other forces - or combinations of forces.]

Fig. 1a shows the various lengths involved.

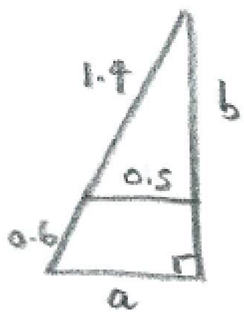

Fig. 1a

Considering the external forces on the ladder (including the rope, but excluding the platform) (Fig. 2),

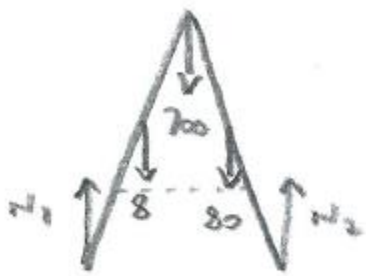

Fig. 2

$N_1 + N_2 = 788$ (1)

Also, taking moments about the base of the righthand side of the ladder :

$-N_1(2a) + (8)\left(\frac{3a}{2}\right) + 700a + 80\left(\frac{a}{2}\right) = 0$ (2),

so that $N_1 = \frac{1}{2}(12 + 700 + 40) = 376$, and hence $N_2 = 412$

Fig. 3 shows the force diagram for the lefthand side of the ladder (excluding the rope and the platform), where X & Y are the components of the reaction from the righthand side of the ladder, and R_1 is the reaction from the platform and person.

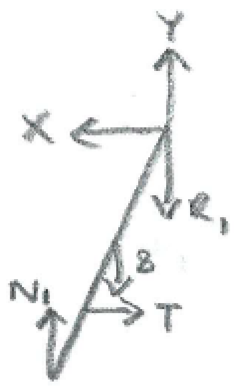

Fig.3

Horizontally this gives $T = X$ (3)

and vertically: $N_1 + Y = 8 + R_1$,

As $N_1 = 376,\ 368 + Y = R_1$ (4)

By N3L, the reaction forces on the righthand side of the ladder from the lefthand side are X & Y, as shown in Fig.4. And R_2 is the reaction from the platform and person.

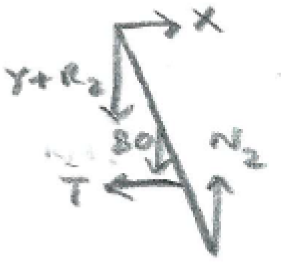

Fig. 4

Then, vertically: $N_2 = 80 + Y + R_2$

As $N_2 = 412,\ 332 = Y + R_2$ (5)

(Horizontally just reproduces $T = X$)

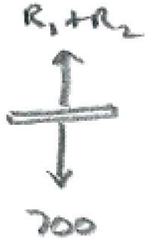

Fig. 5

Fig. 5 shows the forces on the platform and person, giving

$R_1 + R_2 = 700$ (6)

[Note that this could have been obtained by adding (4) & (5).]

In order to involve the location of T, we can take moments about the top of the ladder, for say the lefthand side, to give:

$-N_1 a + Tb + (8)\left(\frac{a}{2}\right) = 0$ (6), so that $T = \frac{a}{b}(376 - 4) = \frac{372a}{b}$

Also, from Fig. 1a, by similar triangles, $\frac{a}{0.5} = \frac{2}{1.4}$, so that $a = \frac{5}{7}$,

and $b^2 = 1.4^2 - 0.5^2$, so that $b = \frac{\sqrt{171}}{10} = \frac{3\sqrt{19}}{10}$,

and hence $T = \frac{372(5)(10)}{7(3\sqrt{19})} = 203.197$