Forces – Q6 [13 marks](2/6/21)

Exam Boards

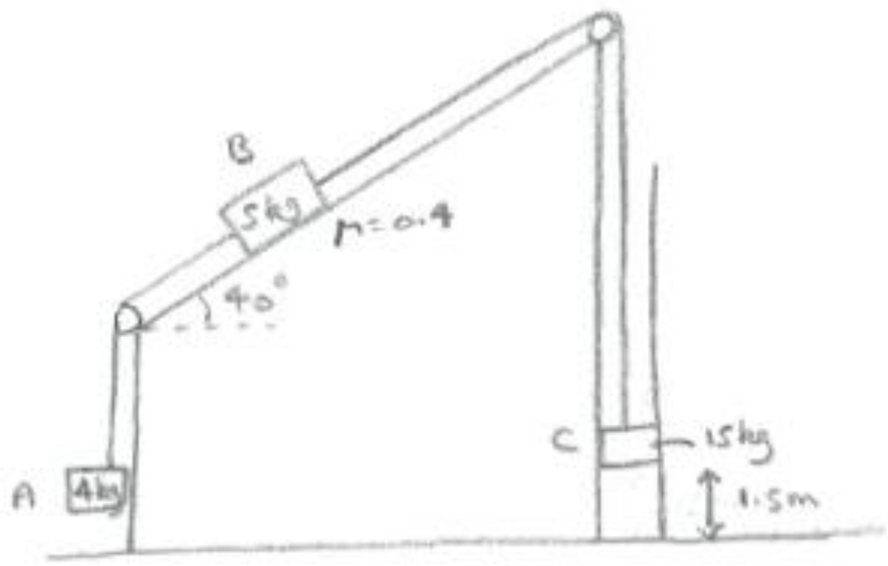
OCR : Mechanics (Year 1)

MEI: Mechanics a

AQA: Mechanics (Year 1)

Edx: Mechanics 1 (Year 1)

Referring to the diagram below, A has mass $4kg$ and hangs freely, B has mass $5kg$ and moves on a rough slope which makes an angle $40°$ with the horizontal, with a coefficient of friction of 0.4, and C has mass $15kg$ and slides in a groove, against a constant frictional force of $15N$. The objects are connected by light inextensible ropes, which pass over smooth pegs. Initially C is $1.5m$ above the ground.



The system is released from rest. Find how far up the slope B travels. It can be assumed that A and B don't reach the next peg, and that the rope connecting A and B remains taut. Also, air resistance can be ignored. [13 marks]

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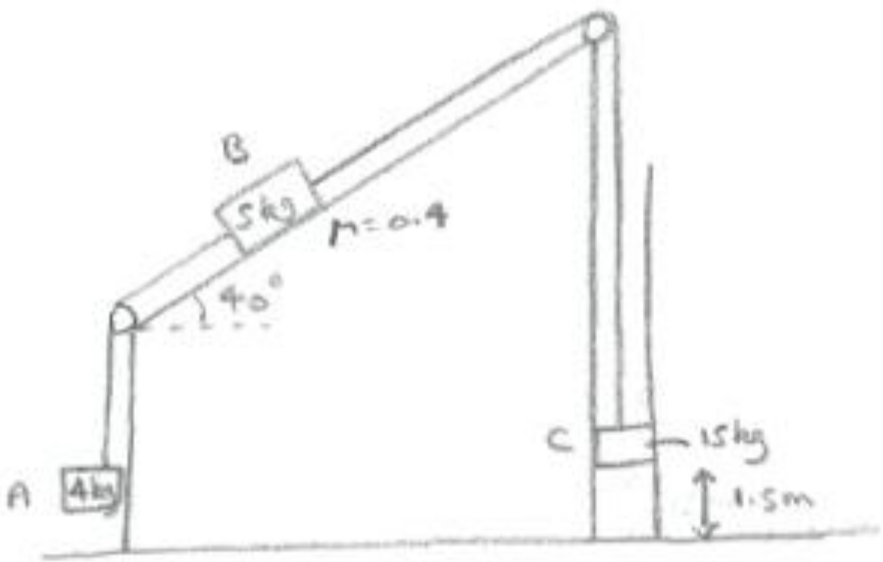

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Solution

Method 1: N2L & suvat approach

Until C hits the ground:

Applying N2L to A: $T_{AB} - 4g = 4a$ (1) [1 mark]

Applying N2L to B:

$T_{BC} - T_{AB} - 5gsin40° - \mu(5gcos40°) = 5a$ (2) [1 mark]

Applying N2L to C: $15g - T_{BC} - 15 = 15a$ (3) [1 mark]

Then (1) + (2) + (3) gives

$-4g - 5gsin40° - (0.4)(5gcos40°) + 15g - 15 = 24a,$

so that $a = \frac{9.8}{24}(11 - 5sin40° - 2cos40°) - \frac{15}{24} = 1.92871ms^{-2}$

[2 marks]

Speed of B after travelling $1.5m$:

$'v^2 = u^2 + 2as' \Rightarrow v^2 = 2(1.92871)(1.5) = 5.78613$

$\Rightarrow v = 2.40544ms^{-1}$ (*) [2 marks]

Once C hits the ground, the rope BC becomes slack, and the N2L eq'ns become:

A: $4g - T_{AB}' = 4a'$ $(1')$ [1 mark]

(where a' is now the acceleration towards the ground)

B: $T_{AB}' + 5gsin40° + \mu(5gcos40°) = 5a'$ $(2')$ [1 mark]

(noting that friction is still acting to oppose motion up the slope)

Then $(1')$+$(2')$ gives

$4g + 5gsin40° + \mu(5gcos40°) = 9a',$

so that $a' = \frac{9.8}{9}(4 + 5sin40° + (0.4)5cos40°) = 9.52345ms^{-2}$

[2 marks]

Then, from (*), $'v^2 = u^2 + 2as' \Rightarrow 0 = 2.40544^2 + 2(-9.52345)s,$

so that the total distance moved up the slope is

$1.5 + \frac{2.40544^2}{2(9.52345)} = 1.80378 = 1.80m\ (3sf)$ [2 marks]

Method 2: Energy approach

[KE is dissipated by C on hitting the ground, and in order to determine this KE the tension in the rope BC is needed (which can be obtained from the start of the N2L and suvat approach).

To avoid this complication, it would be necessary to consider the motion up to the point where C hits the ground, and use an energy approach to find the speed of B at this point. Then, with the rope BC being slack, an energy approach can be used again to find the further distance moved by B.]

Once C hits the ground, the rope BC becomes slack, and B continues to move up the slope. Let d be the total distance moved by B.

Then total loss of GPE is

$15g(1.5) - 4gd - 5g(dsin40°) = 220.5 - 70.69659d$

And total work done against friction is

$\mu(5gcos40°)d + 15(1.5) = 15.01447d + 22.5$

The KE of A & B is zero at the start and end, but C has KE that has been dissipated:

KE of C when it hits the ground is

Original GPE of C less work done against friction, less work done against T_{BC} (from N2L & suvat approach)

From (3) in the N2L & suvat approach,

$15g - T_{BC} - 15 = 15a$, and $a = 1.92871$,

so that $T_{BC} = 103.06935$,

and KE of C when it hits the ground is

$15g(1.5) - 15(1.5) - 103.06935(1.5) = 43.39598\,J$

Then total loss of GPE equals total work done against friction plus KE dissipated,

so that $220.5 - 70.69659d = 15.01447d + 22.5 + 43.39598$,

and $d = \frac{154.60402}{85.71106} = 1.80378m$