Fictitious Forces (5 pages; 25/10/20)

(1) Newton's laws of motion are only valid in what is called an **inertial frame of reference**: one that is not accelerating (and therefore not rotating).

[In the following notes, "frame of reference" will be abbreviated to "frame".]

On the assumption that the Sun is not accelerating through space, the inertial frame can be taken to be relative to the Sun.

As the Earth is accelerating through space relative to the Sun (according to Kepler's 2nd law, planets sweep out equal areas in equal intervals of time, and this implies that they speed up and slow down in their orbits), and is also rotating, it is not an inertial frame (though for most purposes it can be assumed to be, approximately).

(2) In order to investigate the motion of an object, we need to establish the forces on it relative to an inertial frame (a force diagram can then be drawn, and Newton's 2nd law used to create equations, which can then be solved).

(3) Example 1: Consider a passenger on a train, who is facing in the direction of motion. Suppose that there is a glass on the table in front of him, and that the table is completely smooth.

If the train is accelerating, then the glass will move towards the passenger (who himself will be pressed against the seat).

Relative to the (approximate) inertial frame of the Earth, the glass has no force applied to it in the plane of the table. But in the frame of the train, there is an **apparent force** on the glass, causing it to move towards the passenger.

The term **fictitious force** is also used to describe this phenomenon.

Note that, despite the use of the words 'apparent' and 'fictitious', the effect on the glass is nevertheless very real, from the point of view of the passenger.

(4) Example 2: A fairground 'wall of death', where an individual stands against a wall, inside a drum which is rotated. The speed of rotation is increased, and at a suitable speed it is possible to remove the floor, leaving the individual pinned to the wall.

In this motion, the normal reaction of the wall on the individual provides the centripetal force which gives rise to circular motion.

In the (approximate) inertial frame of the Earth, the forces on the individual are:

- his weight

- the normal reaction from the wall

- friction up the wall (proportional to the normal reaction)

The vertical forces are balanced, and the net horizontal force (ie the normal reaction) is a centripetal force, giving rise to circular motion.

By contrast, in the non-inertial frame of the wall, the forces on the individual are:

- his weight

- the normal reaction from the wall

- friction up the wall

- an apparent '**centrifugal force**'

This extra force is needed to account for the fact that the individual is not moving relative to the wall. In the horizontal direction, he feels two opposing forces: an apparent force, pressing him onto the wall and the normal reaction from the wall, balancing this apparent force.

This is the same situation as for the passenger on the train, who feels pressed against his seat, and feels the seat pressing against him.

Thus the apparent centrifugal force is equal in magnitude to the normal reaction, which in turn is equal to the centripetal force.

Note that the apparent force is being introduced in order for Newton's 2nd law to be satisfied (apparently), given that there is no net acceleration relative to the wall. In reality, Newton's 2nd law ought not to be applied, because the rotating wall is not an inertial frame.

(5) Centrifugal and other fictitious forces have generally been airbrushed out of Mechanics books now. One book mentions "the force that used to be referred to as the centrifugal force" (shades of the "Knights of Ni" in Monty Python's Holy Grail).

(6) Example 3: An object on the Earth's surface is subject to the following forces relative to the inertial frame of the Sun:

- gravity of the Earth

- normal reaction from the ground

- gravity of the Sun/Moon etc

The net effect of these forces causes the object to

(a) rotate about the centre of the Earth (along with the surface of the Earth itself)

(b) accelerate (with the Earth) about the Sun

Only part of the gravity of the Earth is needed to cause the rotation of the object about the centre of the Earth. The remainder is balanced by the normal reaction of the ground. Were gravity to be very feeble, but the Earth to be spinning at the same rate, there would be a tendency for the object to fly off into space.

Ignoring (b), Newton's 2nd law gives $mg - R = \frac{mv^2}{r}$ (1)

(with usual notation)

By contrast, relative to the non-inertial frame of the ground, the forces on the object are as before, but with the addition of two fictitious forces: the centrifugal force and another apparent force, called the **'Coriolis'** force, which arises from any motion of the object relative to the ground.

Ignoring (b) again, and also the Coriolis force, the artificial application of Newton's 2nd law in the non-inertial frame gives

$mg' - R = 0$ (2)

where g' is the **apparent gravity** on the object.

(1) and (2) $\Rightarrow mg' = R = mg - \frac{mv^2}{r} \Rightarrow g' = g - \frac{v^2}{r}$

[An individual would feel the gravitational force to be lighter, compared to an (admittedly hypothetical) situation where the Earth is not spinning. Part of the true gravitational force

($g = g' + \frac{v^2}{r}$) can be thought of as being used to maintain the individual in his circular motion round the Earth.]

In a more general case, where there is variable rotation, another fictitious force, the 'Euler' force, comes into play.

(7) In Einstein's General Theory of Relativity, even gravity itself can be interpreted as a fictitious force. This is suggested by the fact that the gravitational force, in common with the fictitious forces described so far, is proportional to mass (creating the

arguably strange state of affairs that the Earth exerts different forces on different objects). Thus the laws of Physics can be considered only to apply in empty space, with an adjustment needed for the 'fictitious' force of gravity, if we insist on using a frame of reference near a massive object.