Discrete Random Variables Q4 [Problem/H](9/6/21)

If $X \sim B(n, p)$, prove that $E(X) = np$

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Solution

$$E(X) = \sum_{x=0}^{n} \binom{n}{x} p^x (1-p)^{n-x} x$$

$$= \sum_{x=1}^{n} \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x} x \quad \text{(as the 1st term vanishes)}$$

$$= np \sum_{x=1}^{n} \frac{(n-1)!}{(x-1)!(n-x)!} p^{(x-1)} (1-p)^{n-x}$$

$$= np \sum_{x-1=0}^{n-1} \frac{(n-1)!}{(x-1)!(n-x)!} p^{(x-1)} (1-p)^{n-x}$$

Let $u = x - 1$ and $N = n - 1$

Then $E(X) = np \sum_{u=0}^{N} \frac{N!}{u!(N-u)!} p^u (1-p)^{N-u}$

$$= np \sum_u P(X = u) = np$$