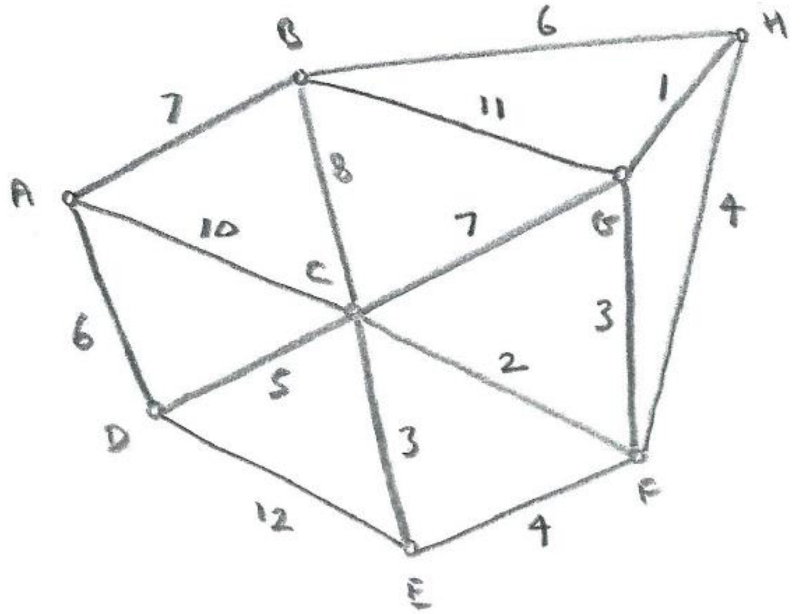
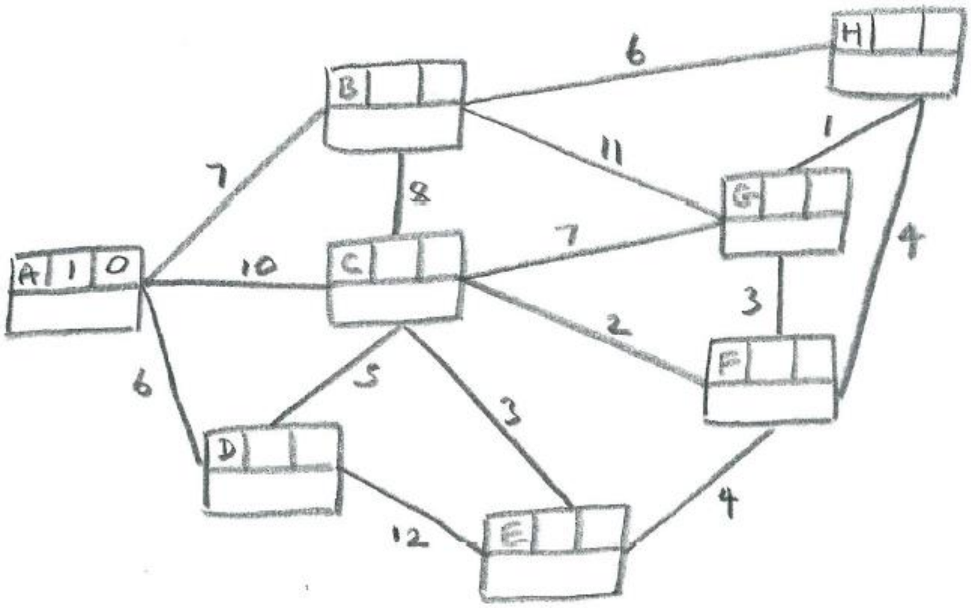
Dijkstra's Algorithm (5 pages; 18/11/20)

The problem is to find the shortest path between two specified nodes. Dijkstra's algorithm in fact finds the shortest distances from a specified node to each of the other nodes in the network.

Example



We will find the shortest distances from A to each of the other nodes. First of all, we replace the nodes with boxes that will display the working of the algorithm:



The contents of the boxes will be explained as we go along.

The aim is to establish provisional (or 'temporary') distances for the nodes, and then find improvements where possible.

We start by attaching ‘temporary labels' to the nodes directly connected to A; ie B, C & D. These labels are just the lengths of AB, AC & AD; ie 7, 10 & 6 (they are placed in the bottom of the boxes).

We can now be sure that the shortest distance from A to D is 6, as any indirect route from A to D would have to pass through either B or C, and AB & AC both have lengths of at least 6. Thus D can be given a ‘permanent label' of 6 (this goes in the right-hand cell at the top of the box). As D is the 2nd node to be given a permanent label (given that A has a permanent label of 0), a 2 is placed in the middle cell at the top of its box.

We now investigate whether any of the remaining temporary distances can be improved on by considering routes leading directly from D.

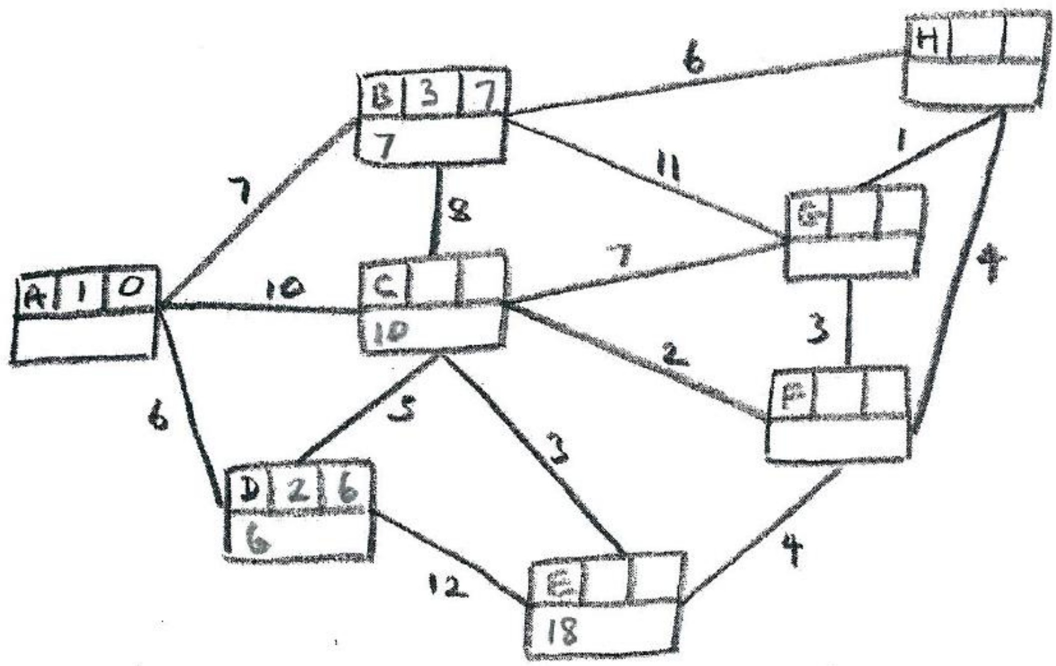
For C, we note that $6 + 5 > 10$, so that no improvement is possible. For E, we are able to create a temporary label of

$6 + 12 = 18$ (representing the route ADE). Note that a shorter route would be $ADCE$, but the algorithm cannot pick this up straightaway.

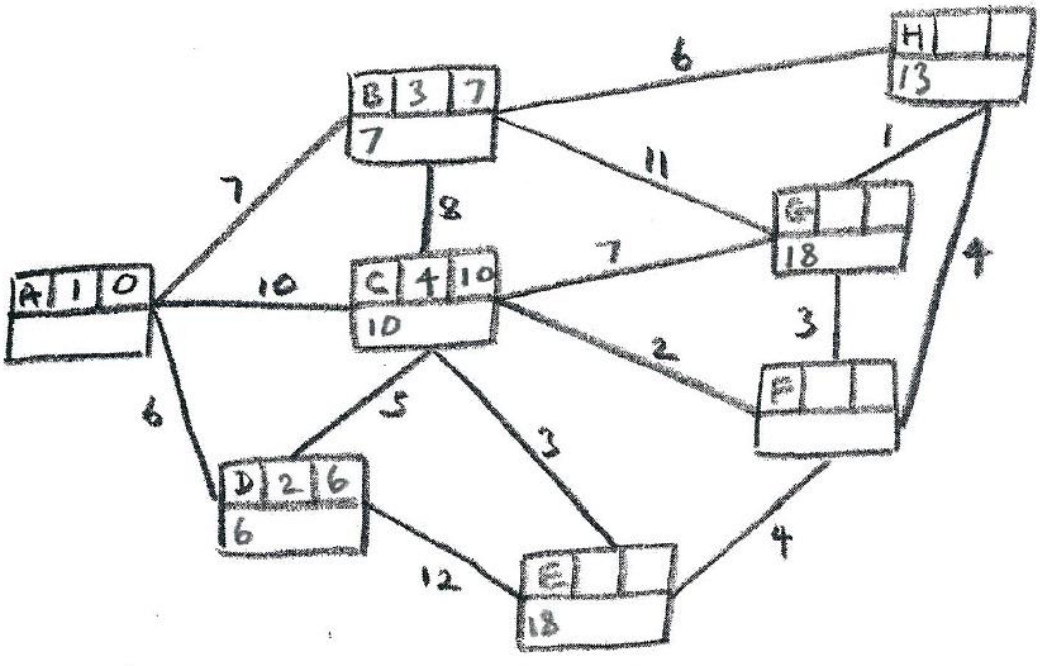
We now see that the smallest of the remaining temporary labels is 7 (for B). This indicates that 7 is the shortest possible distance from A to B.

The reasoning is as follows: Consider the shortest route from A to B, and let X be the last node on this route that (so far) has a permanent label. (There must be such a node, as A is permanently labelled.) Then let Y be the node after X (on this route). Y of course will have a temporary label. Clearly B is a possible candidate for Y, with X being A. And having any other node as Y would result in a longer distance from A to B, as Y would have a bigger temporary label than B. So Y has to be B, and thus B is the 3rd node whose label is made permanent.

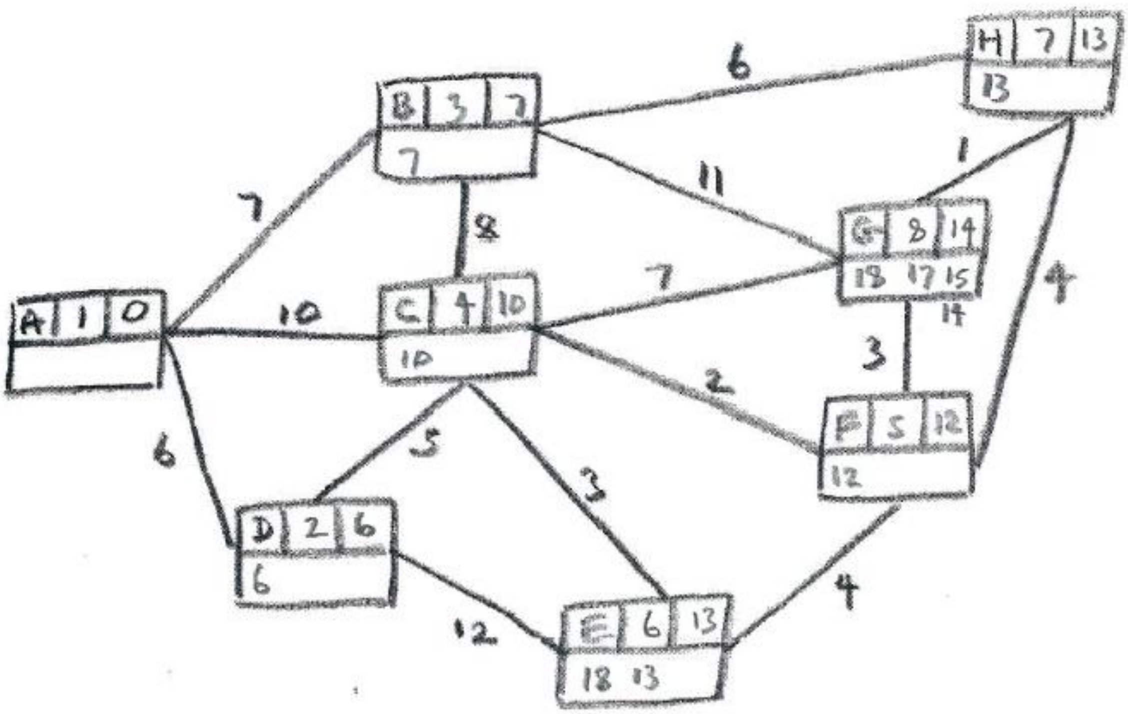
The network now appears as follows:



Repeating this procedure, we find that C is the 4th node whose label is made permanent, and the network becomes:



After continuing this process, we arrive at the final network:



Notes

(i) If more than one node shares the smallest temporary label, it doesn’t matter which is chosen.

(ii) If we wish to establish which of (say) AH, BH & CH has the shortest possible distance, then we can carry out Dijkstra's algorithm with H as the starting node.