Differentiation – Q3 [9 marks](22/5/21)

Exam Boards

OCR : AL (Year 1)

MEI: AL (Year 1)

AQA: AL (Year 1)

Edx: AL (Year 1)

Find the turning points of $y = (x^2 - 4x + 3)^2$, and hence sketch the curve. [9 marks]

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Solution

Method 1

As $x^2 - 4x + 3 = (x - 1)(x - 3)$,

$y = (x - 1)^2(x - 3)^2$ [1 mark]

Then $\frac{dy}{dx} = 2(x - 1)(x - 3)^2 + (x - 1)^2(2)(x - 3)$

$= 2(x - 1)(x - 3)(x - 3 + x - 1)$

$= 4(x - 1)(x - 3)(x - 2)$ [2 marks]

$\frac{dy}{dx} = 0$ when $x = 1, 2$ & 3 [1 mark]

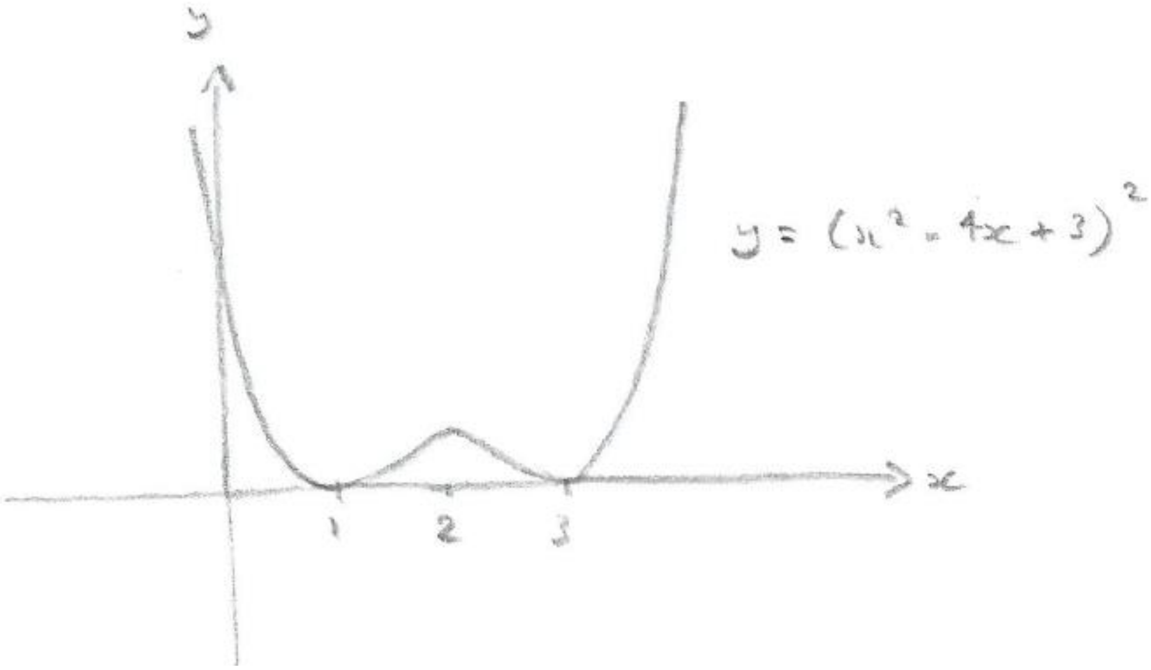
At $x = 1$, $\frac{dy}{dx}$ changes from –ve to +ve, indicating a min. point.

At $x = 2$, $\frac{dy}{dx}$ changes from +ve to -ve, indicating a max. point.

At $x = 3$, $\frac{dy}{dx}$ changes from –ve to +ve, indicating a min. point.

[2 marks]

The min. points are therefore (1, 0) and (3, 0), whilst the max. is at (2,1). [1 mark]



[2 marks]

Method 2

$(x^2 - 4x + 3)^2 \geq 0$ and $(x^2 - 4x + 3)^2 = (x-1)^2(x-3)^2 = 0$ has roots at $x = 1$ & 3, so that there are minima at these two points.

For $x = 1 - t$, $y = x^2 - 4x + 3$ and hence y $= (x^2 - 4x + 3)^2$ increases as t increases, and similarly for $x = 3 + t$.

For $1 < x < 3$, y $= (x^2 - 4x + 3)^2$ attains a max. when

$x^2 - 4x + 3$ (which is negative in this range) is at a min. ; ie when

$x = 2$.