Differentiation Q2 – Practice/Y2/M (22/5/21)

(i) Find $\frac{d}{dx}(x^x)$

(ii) Show that $\frac{d}{dx} log_a x = \frac{1}{xlna}$

(iii) Find $\frac{d}{dx}\left(x^{sinx}\right)$

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Solution

(i) Let $y = x^x$

Then $lny = xlnx$

and $\frac{1}{y}\frac{dy}{dx} = lnx + x(\frac{1}{x})$

so that $\frac{dy}{dx} = y(lnx + 1) = x^x(1 + lnx)$

(ii) $\frac{d}{dx}log_a x = \frac{d}{dx}(log_a e.\, lnx) = \frac{1}{xlna}$ (as $log_a b = \frac{1}{log_b a}$)

(iii) $\frac{d}{dx}\left(x^{sinx}\right) = \frac{d}{dx}\left(e^{lnx.sinx}\right) = e^{lnx.sinx}(\frac{1}{x}sinx + lnx.\, cosx)$

$= x^{sinx}(\frac{1}{x}sinx + lnx.\, cosx)$

(iv) **Method 1**

Let $a = e^b$. Then $\frac{d}{dx}(a^x) = \frac{d}{dx}(e^{bx}) = be^{bx} = lna.\, a^x$

Method 2

Let $y = a^x$. Then $lny = xlna,$

and, differentiating wrt x gives $\frac{1}{y}\frac{dy}{dx} = lna$, so that $\frac{dy}{dx} = lna.a^x$