Differential Equations – Q2 [Practice/E](12/7/21)

Solve $sinx \frac{dy}{dx} + secx.y = cosx$

Solution

Before we find an integrating factor, can we rearrange the LHS into the form $P(x)\frac{dy}{dx} + P'(x)y$?

Dividing through by $cosx$ gives

$tanx\frac{dy}{dx} + sec^2x.y = 1,$

so that $\frac{d}{dx}(ytanx) = 1$

and hence $ytanx = x + C,$

so that $y = (x + C)cotx$

Note: Finding the IF here is quite time-consuming:

First of all, $\frac{dy}{dx} + \frac{1}{cosxsinx}.y = cotx$

Then IF = $\exp\{\int \frac{1}{cosxsinx}dx\}$

$I = \int \frac{1}{cosxsinx}dx = 2\int \frac{1}{sin2x}dx$

$= 2\int \frac{sin2x}{sin^2 2x}dx = 2\int \frac{sin2x}{1-cos^2 2x}dx$

Let $u = cos2x$, so that $du = -2sin2x\ dx$

and $I = -\int \frac{1}{1-u^2}\ du = -\frac{1}{2}\int \frac{1}{1-u} + \frac{1}{1+u}\ du$

$= -\frac{1}{2}\{-\ln|1-u| + \ln|1+u|\}$

$= \frac{1}{2}\ln\left|\frac{1-u}{1+u}\right| = \frac{1}{2}\ln\ \left|\frac{1-cos2x}{1+cos2x}\right|$

$= \frac{1}{2}\ln\left|\frac{2sin^2x}{2cos^2x}\right| = \frac{1}{2}\ln(tan^2x) = \ln\ |tanx|$

So $IF = \exp\{\ln|tanx|\} = tanx$