Differential Equations – Q1 [Practice/E](26/5/21)

Find the general solution of the differential equation

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Solution

The auxiliary equation is $\lambda^2 - 6\lambda + 9 = 0$ or $(\lambda - 3)^2 = 0$, which has a repeated root of $\lambda = 3$.

The complementary function is $y = (A + Bx)e^{3x}$

The particular integral will be of the form $y = asin2x + bcos2x$.

Substituting into (*) gives

$$(-4asin2x - 4bcos2x) - 6(2acos2x - 2bsin2x)$$

$$+9(asin2x + bcos2x) = cos2x$$

Equating coefficients of $sin2x$ and $cos2x$:

$$-4a + 12b + 9a = 0 \text{ and } -4b - 12a + 9b = 1$$

leading to $a = -\frac{12}{169}$ and $b = \frac{5}{169}$

The general solution is therefore

$$y = (A + Bx)e^{3x} - \frac{12sin2x}{169} + \frac{5cos2x}{169}$$