Critical Path Analysis - Part 2 (14 pages; 7/11/20)

Resources and Scheduling

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(1) Introduction

(1.1) Important considerations for a project are usually:

(a) minimising the completion time

(b) minimising the number of workers

(c) minimising costs

(1.2) There are three charts that can be created to help with resource decisions:

Gantt (or Cascade) chart

Resource histogram

Scheduling diagram

(1.3) Possible complications that may be introduced:

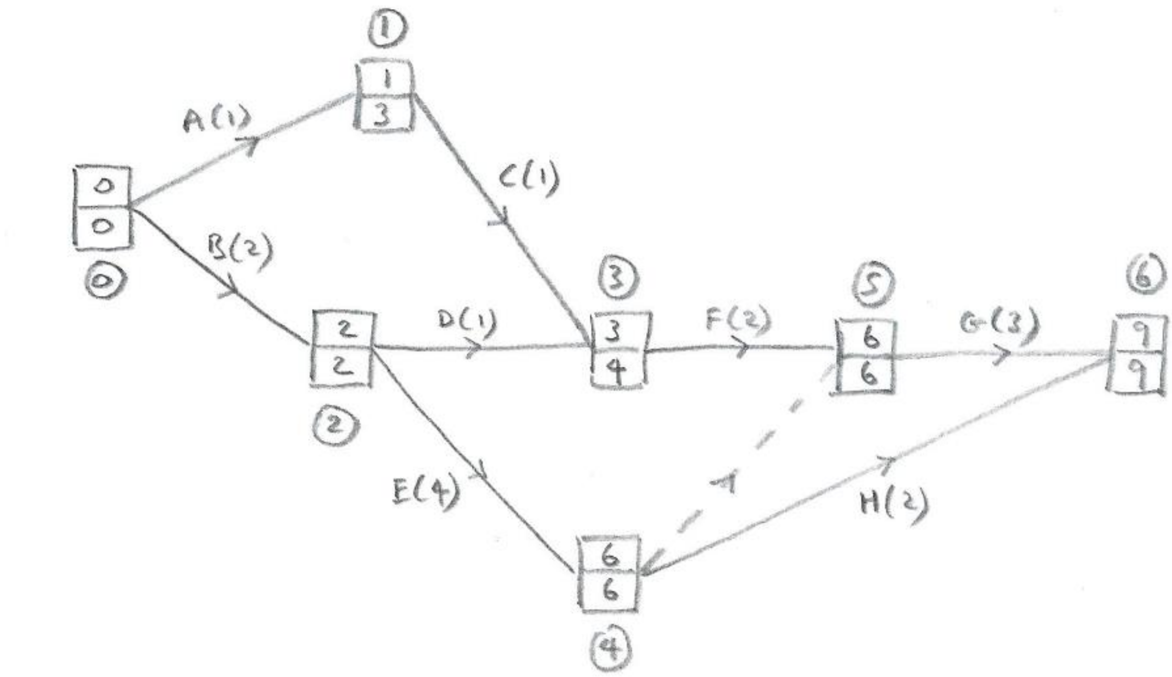
(i) Activities can't take place at the same time (eg if there is only one machine or worker available for those activities).

(ii) An activity can't take place until a specified time.

Any changes to activities are liable to alter the critical path(s), and the activity network may need to be redrawn.

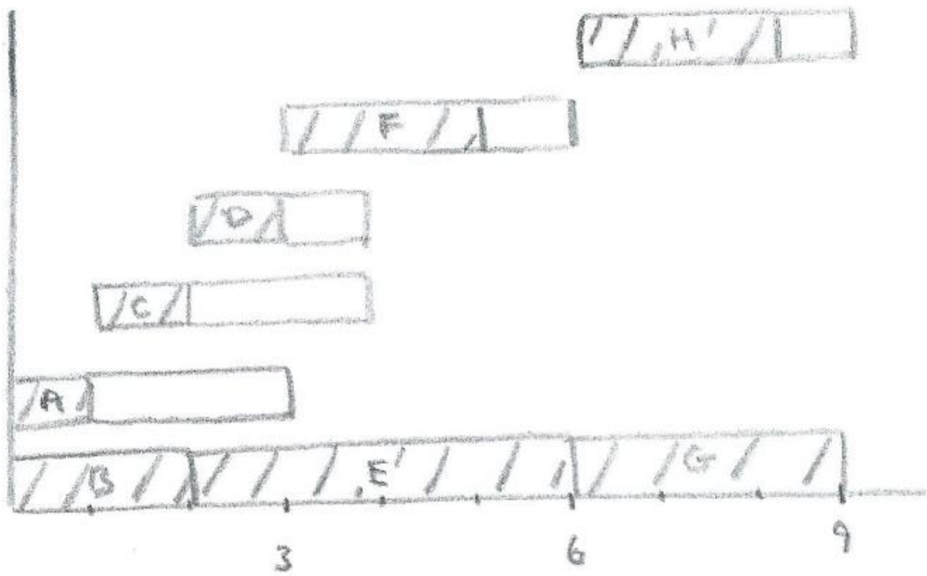
(2) Gantt (or Cascade) Chart

(2.1) Example: Consider the following activity network (using activity-on-arc):



(The critical path is BEG.)

The corresponding Gantt Chart is shown below.



(2.2) Notes

(i) The critical activities may appear in one row (as in the diagram), but are sometimes given separate rows. There may be more than one critical path.

(ii) Each of the non-critical activities is assumed to start at the earliest possible time; eg for F, the earliest event time for node 3 (which is also referred to as the earliest start time for F).

(iii) The shaded sections of the bars are the durations of the activities, with the floats being unshaded. (Sometimes there is no shading, and the float is indicated by a dotted border.)

(iv) The number of workers required for each activity is sometimes shown on the bar.

(v) A Gantt chart can be used to establish which activities definitely have to be taking place at a given point in time (ie activities that can't be shuffled away from a vertical line at that point).

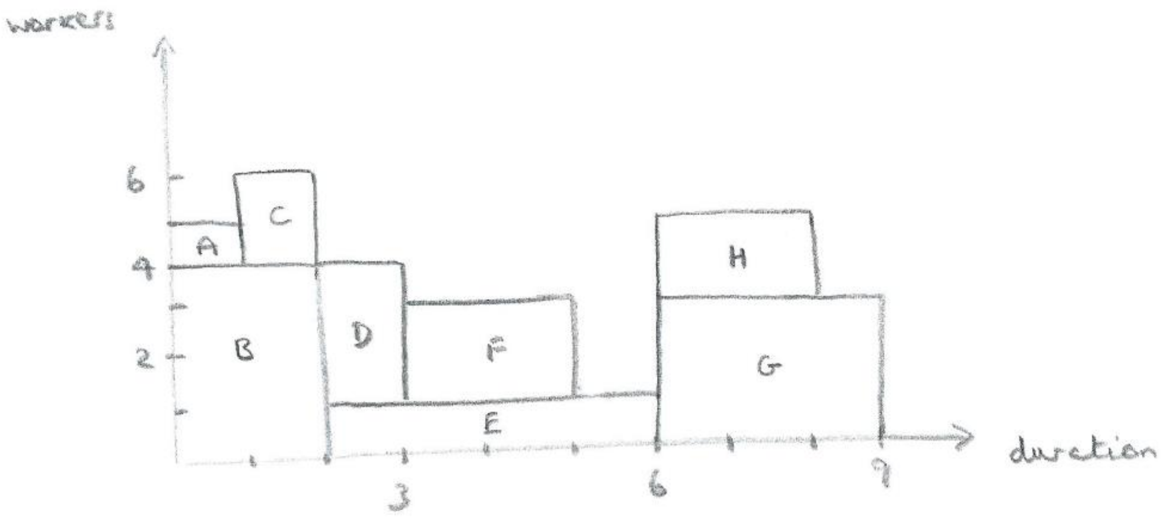
(vi) The activity precedences are not recorded in the diagram (eg the fact that C has to take place before F). For this reason, the Gantt Chart may have limited use, and it may be better to work from the activity network instead.

(3) Resource Histogram & Resource Levelling

(3.1) The numbers of workers needed for the activities in the above example are shown in the table below.

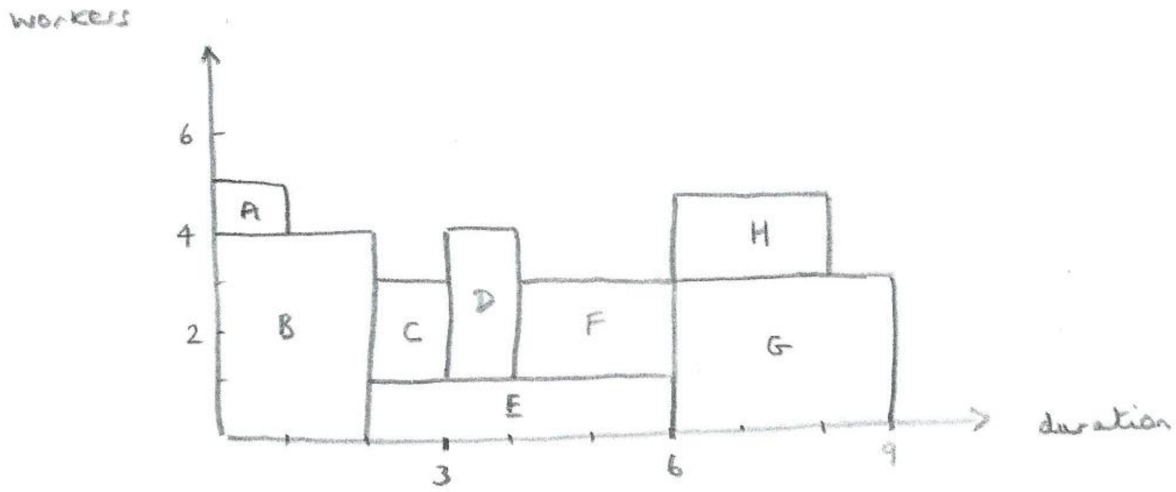
Activity	A	B	C	D	E	F	G	H
Workers	1	4	2	3	1	2	3	2

This enables the Resource histogram to be constructed, as below.



As can be seen, the width of each activity is the same as in the Gantt chart.

(3.2) Sometimes it is possible to shuffle activities along (within their boxes in the Gantt chart), in such a way as to reduce the total number of workers required at any one time (without increasing the completion time). This then leads to a revised resource histogram (see below for the example being considered). Here we see that 5 workers are needed.



Because the activity precedences are not recorded in the Gantt chart, we have to be careful that the revised Resource histogram doesn't infringe those precedences.

In a simple case, where only one worker is needed for each activity, the activities in the Gantt chart can often be shuffled visually. For example, if it is possible to shuffle the activities so that no vertical line crosses more than 3 activities, then 3 workers will be sufficient. (As mentioned though, we have to be careful that precedences are not infringed. It can be easier to check precedences from the activity network than from the original precedence table.)

(3.3) A lower bound for the smallest possible number of workers needed can be found by considering the ideal situation in which the activities are shuffled into a rectangle, the base of which is the minimum completion time.

For the current example, the area of the rectangle is the number of man-days needed:

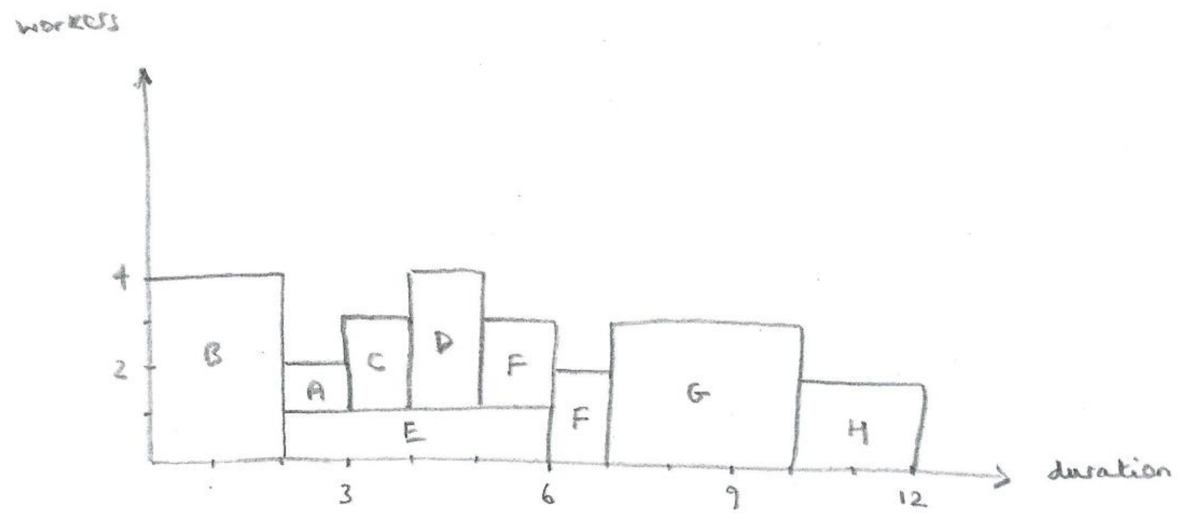
$1 + 8 + 2 + 3 + 4 + 4 + 9 + 4 = 35$,

where the number of man-days for B, for example, is $2 \times 4 = 8$

Thus the lower bound is $\frac{35}{9} = 3.89$ (3sf); ie 4 workers, as a whole number is required.

Also, 4 workers are required for activity B, which in general places a further constraint on the lower bound.

(3.4) It may be the case that only 4 workers are available (in this example), and the completion time has to be extended. The Resource histogram can then be adapted to achieve this (see diagram below). Note that some activities will move outside their boxes in the Gantt chart, and that the precedences will need to be taken into account.



Note that activity F is in two parts (this avoids having an empty space below the 2nd half of F).

(3.5) Resource Histogram Summary

[More than 1 worker may be needed for each activity.]

(i) Determine a lower limit for the number of workers needed.

(ii) Use resource levelling to find the minimum number of workers needed.

(iii) Modify the resource histogram to find the completion time if a limited number of workers are available.

(iv) Check precedences!

(4) Scheduling Diagram

(4.1) Each worker is allocated a row containing their activities.

It will usualy be the case that only one worker is required for each activity.

(4.2) 'Situation A': There is no limit on the number of workers available. The scheduling diagram shows the minimum number of workers needed to finish the project within the minimum completion time.

Standard procedure:

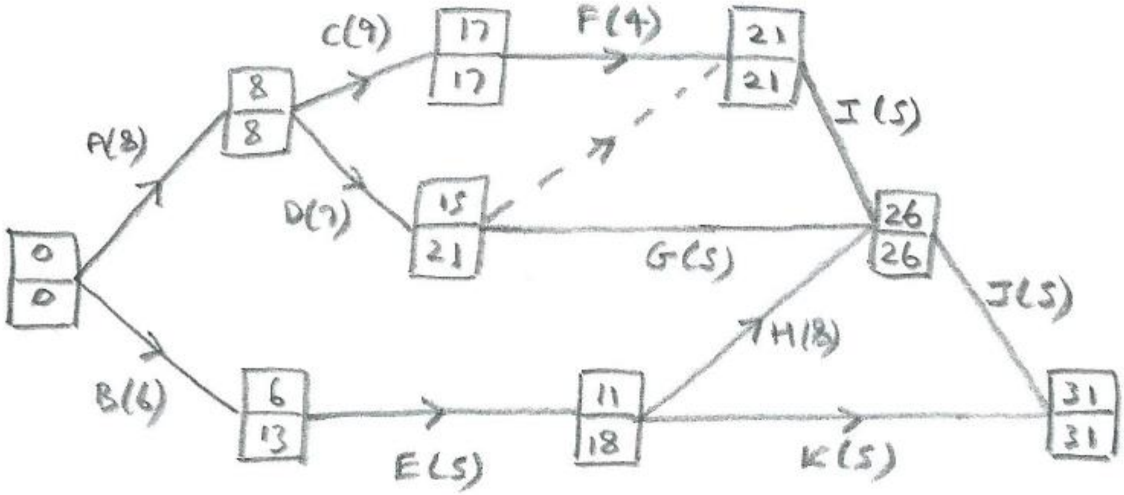
(i) Allocate the critical activities to the 1st worker (who will have no time available for further activities, as the project is to be completed in the minimum completion time).

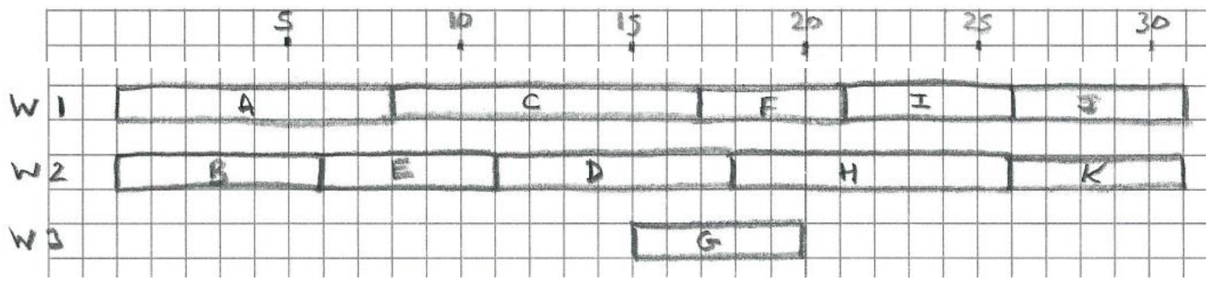
(ii) From the Gantt chart, list the remaining activities in order of increasing earliest start time. If two activities have the same earliest start time, the one with the lowest latest finish time is chosen first (for maximum flexibility).

(iii) Add a new worker if an activity would otherwise be prolonged beyond its latest finish time.

(iv) Ensure throughout that the activity precedences are not contravened.

Example (Situation A)





(4.3) 'Situation B': Only a limited number of workers are available. The scheduling diagram shows the new completion time.

Standard procedure:

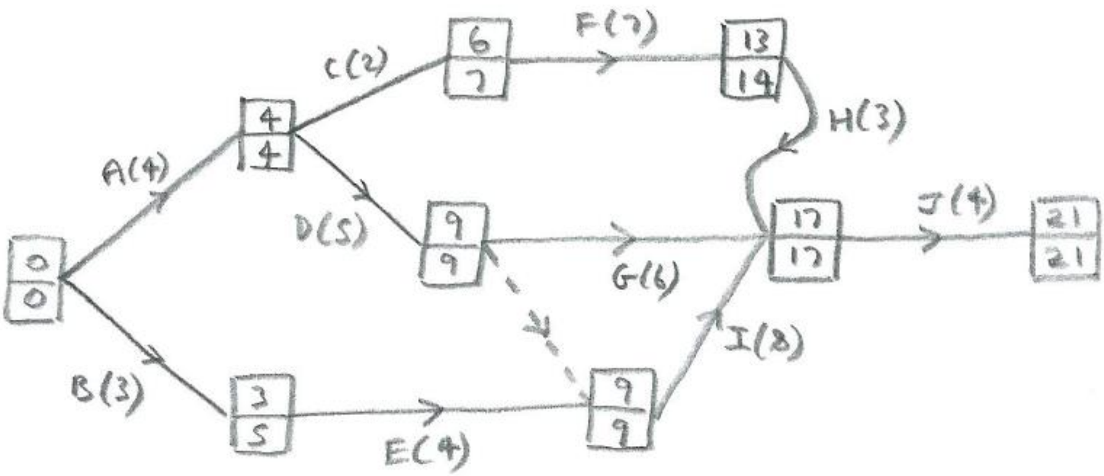
(i) Allocate a row to each of the workers.

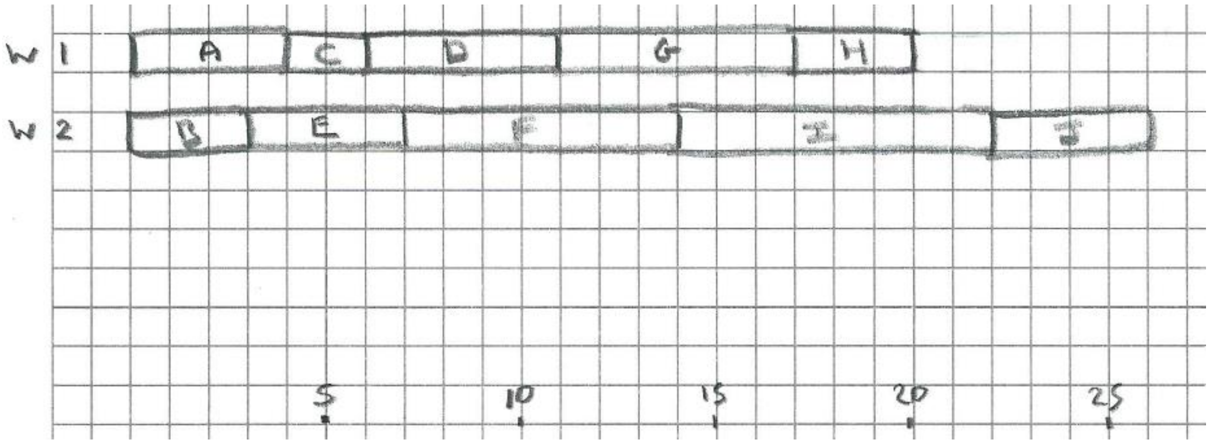
(ii) Treat all activities (ie including critical activities) in the same way, listing them (as before) in order of increasing earliest start time (and if two activities have the same earliest start time, the one with the lowest latest finish time is placed first).

(iii) As each worker completes an activity, allocate the next activity in the list to them, but taking account of any precedences.

Note: If activities have to be delayed, then the original earliest start times and latest finish times may become invalid (but this tends to be glossed over; the important thing is to take account of any precedences).

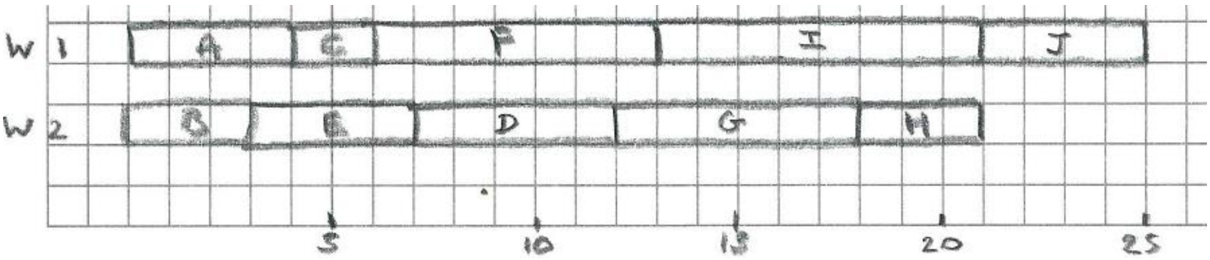
Example (Situation B) (time is measured in days)



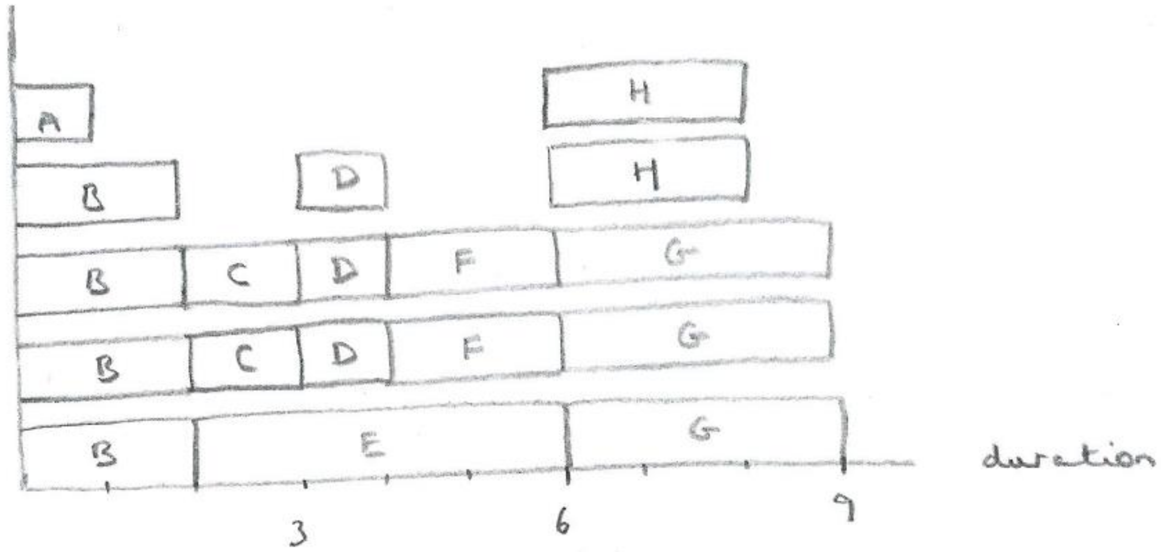


(4.4) The standard scheduling procedure (where only a limited number of workers are available) doesn't always give the best solution.

For the previous example, it is in fact possible to complete the project (with 2 workers) in 25 days (as shown below).



(4.5) The example considered earlier, when resource levelling was carried out, involved more than one worker for each activity. In that case, a scheduling diagram for 5 workers (with the project being completed in the minimum completion time) can be derived directly from the Resource histogram (after levelling), as shown below.



(5) Crashing a network

This is where the completion time is reduced, at a cost.

(5.1) Possible options:

(i) increase the number of workers; eg if an activity takes 4 days when carried out by 1 worker, it may take 2 days with 2 workers

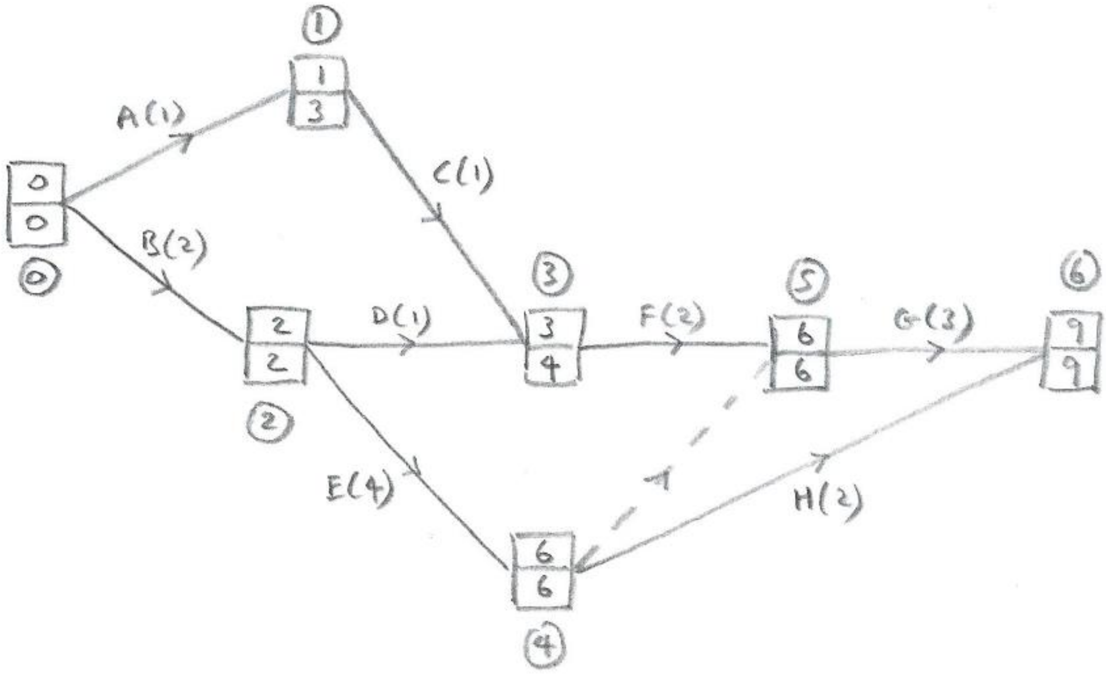
(ii) reduce the duration of an activity by some other expenditure (eg hiring a machine)

(5.2) Issues

(i) The critical activities need to be shortened first.

(ii) Other activities may become critical. (The activity network may need to be re-drawn.)

(5.3) Example



The table below shows the costs associated with the original durations for each activity, for the network considered earlier (shown above). It is possible to reduce some of the durations of the activities, at an extra cost, as shown in the table. If the duration is reduced only partially, then the new cost is calculated on a pro-rata basis; eg for activity E, the cost for 3 days would be £750.

Activity	A	B	C	D	E	F	G	H
Original duration (days)	1	2	1	1	4	2	3	2
Cost (£)	100	300	200	100	500	400	300	300
New	1	1	1	1	2	1	1	1

duration (days)								
Cost (£)	100	500	200	100	1000	600	900	500

Assuming that there are no constraints on the number of workers available, which activities would you recommend be speeded up, if the required reduction in the total time were:

(a) 1 day (b) 2 days (c) 3 days?

And (d) What is the biggest possible reduction, and what is the extra cost associated with this?

Solution

The options for achieving reductions in the durations of activities are:

(1) Reduce B by 1 (extra cost: £200)

(2) Reduce E by 1 (extra cost: £250)

(3) Reduce E by 2 & F by 1 (extra cost: 500+200=£700)

(4) Reduce G by 1 (extra cost: £300)

(5) Reduce G by 2 & H by 1 (extra cost: 600+200=£800)

(a) recommend (1)

(b) recommend (1) & (2)

(c) recommend (1) & (3)

(d) (1), (3) & (5) (reduction of 5, at extra cost of 200+700+800=£1700)

A systematic way of finding the shortest completion time is to create a new activity network, based on all the minimum durations, and then increase the durations of any non-critical activities (ie so that the floats are used up).