

Counting Q7 [Problem/H] (9/6/21)

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Solution

Let the 6 pairs of twins be labelled $AaBb \dots Ff$

Define team 1 to be the team containing A .

Team 1 might, for example, be $AbCd$.

The number of ways of choosing the 3 people to go with A is

$\binom{5}{3}$ [the number of ways of choosing 3 of the 5 remaining pairs]

$\times 2^3$ [as either twin could be chosen for each pair]

For the case where team 1 is $AbCd$, define team 2 to be the team containing E . [Had E been chosen for team 1, it would have been another letter that was chosen to define team 2]

One complete selection of teams would then be:

team 1: $AbCd$

team 2: $a c Ef$

team 3: $B DeF$

There is the following scope for choice:

$\binom{4}{2}$ ways of choosing the 2 people out of $aBcD$ to go in team 2;
combined with the 2 ways of choosing either f or F for team 2

Thus the overall total number of ways is:

$$\binom{5}{3} \times 2^3 \times \binom{4}{2} \times 2 = 10 \times 8 \times 6 \times 2 = 960$$

Note: For team 1, an alternative calculation is as follows:

The number of ways of filling the remaining 3 places in team 1 is $10 \times 8 \times 6$, if order is important; but, as order isn't important, we divide by $3!$, to give 10×8 , as before.