Counting Q4 [Problem/H] (9/6/21)

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Solution

Let X represent A & B.

Suppose that X is to the left of E.

Then C & D can go in the spaces shown here: _X_E_

There are 3 choices for C, and then 2 choices for D, giving 6 possibilities.

Multiply by 2, to include cases where X is to the right of E; giving 12 possibilities.

Multiply by 2 again, as X could be AB or BA, giving 24 possibilities.

Alternative approach

Number of ways with no constraints on C & D, where A is ahead of B (eg XABXX):

4 (ways of placing A)

$\times$ 3! (ways of placing C, D & E)

$= 24$

Including cases where B is ahead of A gives $24 \times 2 = 48$ (1)

Permutations to be excluded, with A ahead of B and C ahead of D:

ABCDE, ABECD, EABCD, CDABE, CDEAB, ECDAB

giving a total of $6 \times 2 \times 2 = 24$ to be excluded (including cases where B is ahead of A and/or D is ahead of C)   (2)

Hence, number of allowable ways $= (1) - (2) = 24$

[It is also possible to consider only situations of the form ABXXX (or BAXXX) and XABXX (or XBAXX), and multiply by 2 to cover the symmetrical situations where we start from the other end.]