

Complex Numbers – Q7 (22/5/21)

Exam Boards

OCR : Pure Core (Year 2)

MEI: Core Pure (Year 2)

AQA: Pure (Year 2)

Edx: Core Pure (Year 2)

Express $(1 - i)^6$ in the form $x + iy$ [5 marks]

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Solution

First of all, express $z = 1 - i$ in modulus-argument form:

By considering the Argand diagram, $|z| = \sqrt{2}$ & $\arg(z) = -\frac{\pi}{4}$

[2 marks]

$$\text{So } z = \sqrt{2}\left(\cos\left(-\frac{\pi}{4}\right) + i\sin\left(-\frac{\pi}{4}\right)\right)$$

Then, by de Moivre's theorem,

$$z^6 = (\sqrt{2})^6 \left(\cos\left(-\frac{6\pi}{4}\right) + i\sin\left(-\frac{6\pi}{4}\right)\right) \quad [1 \text{ mark}]$$

$$= 8 \left(\cos\left(-\frac{3\pi}{2}\right) + i\sin\left(-\frac{3\pi}{2}\right)\right)$$

$$= 8 \left(\cos\left(\frac{\pi}{2}\right) + i\sin\left(\frac{\pi}{2}\right)\right) = 8i \quad [2 \text{ marks}]$$