Complex Numbers Q20 – Practice/E (1/7/21)

Given that $2 - i$ is a root of the equation

$z^4 - 6z^3 - 2z^2 + 50z - 75 = 0$, find the other roots.

Solution

Method 1

$2 + i$ is another root (the conjugate of $2 - i$)

Let the other two roots be α & β.

Then $(2 - i) + (2 + i) + \alpha + \beta = 6;\ \alpha + \beta = 2$

And $(2 - i)(2 + i)\alpha\beta = -75;\ \ 5\alpha\beta = -75;\ \alpha\beta = -15$

So the roots α & β satisfy $x^2 - 2x - 15 = 0$

$\Rightarrow (x - 5)(x + 3) = 0 \Rightarrow x = 5\ or - 3$, and these are the remaining roots.

Method 2

$2 + i$ is another root (the conjugate of $2 - i$)

Write $z^4 - 6z^3 - 2z^2 + 50z - 75$

$= (z - [2 - i])(z - [2 + i])(\ z^2 + bz + c)$

$= (z^2 - 4z + 5)(\ z^2 + bz + c)$,

as $(2 - i) + (2 + i) = 4$ and $(2 - i)(2 + i) = 2^2 + 1^2 = 5$

Then, equating coefficients,

$c = -15$ and $[z^3 :] - 6 = b - 4$, so that $b = -2$

[Check: $[z^2 :]\ -2 = -15 - 4b + 5 \Rightarrow b = -2$]

Thus $z^4 - 6z^3 - 2z^2 + 50z - 75 = (z^2 - 4z + 5)(\ z^2 - 2z - 15)$

And $z^2 - 2z - 15 = 0 \Rightarrow (z - 5)(z + 3) = 0 \Rightarrow z = 5\ or - 3$, and these are the remaining roots.