Complex Numbers Q17– Practice/Y1/E (22/5/21)

Solve the equation $z^2 - 2z + 2 = 0$

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(b) by equating real & imaginary parts

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Solution

(a) $z^2 - 2z + 2 = 0$

$\Rightarrow (z-1)^2 + 1^2 = 0$

$\Rightarrow ([z-1]+i)([z-1]-i) = 0$

$\Rightarrow z = 1 - i \;\; or \;\; 1 + i$

(b) Let $z = a + bi$

Then $(a+bi)^2 - 2(a+bi) + 2 = 0$

$\Rightarrow a^2 - b^2 + 2abi - 2a - 2bi + 2 = 0$

equating real parts: $a^2 - b^2 - 2a + 2 = 0$ (1)

equating imaginary parts: $2ab - 2b = 0$ (2)

(2) $\Rightarrow b(a-1) = 0 \Rightarrow b = 0 \;\; or \;\; a = 1$

From (1), $b = 0 \Rightarrow a^2 - 2a + 2 = 0$

(this can be excluded, as a is real and there are no real solutions to the quadratic equation)

$a = 1 \Rightarrow 1 - b^2 = 0 \Rightarrow b = \pm 1$

Hence $z = 1 \pm i$