Complex Numbers – Q1 [Practice/E](22/6/21)

Represent the following on the Argand diagram:

(i) $|z - i| > |z + 1|$

(ii) $|z - i| = 2|z + 1|$

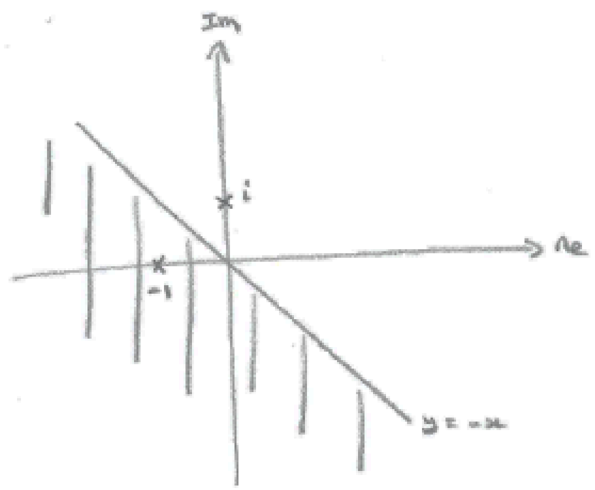
Solution

(i) **Method 1**

Rewriting as $|z - i| > |z - (-1)|$,

z has to be further from i than from -1;

When z is equidistant from these two points, it lies on the perpendicular bisector of the line (segment) connecting the points. So the required region is as shown below.



Method 2

Let $z = x + yi$

Then $|z - i| > |z + 1|$

$\Rightarrow |x + (y-1)i|^2 > |(x+1) + yi|^2$

$\Rightarrow x^2 + (y-1)^2 > (x+1)^2 + y^2$

$\Rightarrow -2y > 2x$

$\Rightarrow y < -x$

(ii) Let $z = x + yi$

Then $|z - i| = 2|z + 1|$

$\Rightarrow |x + (y-1)i|^2 = 4|(x+1) + yi|^2$

$\Rightarrow x^2 + (y-1)^2 = 4\{(x+1)^2 + y^2\}$

$\Rightarrow 3x^2 + 8x + 3y^2 + 2y + 3 = 0$

$\Rightarrow x^2 + \frac{8x}{3} + y^2 + \frac{2y}{3} + 1 = 0$

$\Rightarrow (x + \frac{4}{3})^2 + (y + \frac{1}{3})^2 - \frac{16}{9} - \frac{1}{9} + 1 = 0$

$\Rightarrow (x + \frac{4}{3})^2 + (y + \frac{1}{3})^2 = \frac{8}{9}$