

# Complex Numbers – Q10 (22/5/21)

## Exam Boards

OCR : Pure Core (Year 1)

MEI: Core Pure (Year 1)

AQA: Pure (Year 1)

Edx: Core Pure (Year 1)

Let  $z = \frac{a+i}{1+ai}$ . If  $\arg z = -\frac{\pi}{4}$ , find the possible values of  $a$

[7 marks]

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### Solution

$z$  can be written as  $x - xi$ , where  $x > 0$ , [1 mark]

so that  $(x - xi)(1 + ai) = a + i$  [1 mark]

and  $x + xai - xi + xa = a + i$

Then equating real and imaginary parts:

$x + xa = a$  &  $xa - x = 1$ ; [1 mark]

ie  $x(1 + a) = a$  &  $x(a - 1) = 1$ ,

so that  $x = \frac{a}{1+a} = \frac{1}{a-1}$  [1 mark]

and  $a^2 - a = 1 + a$

$\Rightarrow a^2 - 2a - 1 = 0$  [1 mark]

$\Rightarrow a = \frac{2 \pm \sqrt{8}}{2} = 1 \pm \sqrt{2}$  [1 mark]

Also  $x > 0$ :

$a = 1 \pm \sqrt{2} \Rightarrow x = \frac{1}{a-1} = \frac{1}{\pm\sqrt{2}}$

so that  $a = 1 + \sqrt{2}$  [1 mark]