Clarifications – Pure (3 pages; 2/9/23)

(1) Codomain, image and range

See Pure/Functions – "Mappings & Functions".

(2) Square root

The square root of 4 is ± 2, but $\sqrt{4}$ is (by convention) 2; ie the positive square root.

(3) Prime numbers

0 and 1 are not counted as prime numbers

A prime number can be defined as a number that has exactly 2 distinct factors (and factorisations can't involve 0).

Thus 0 has no factors.

For other numbers, 1 and the number itself will always be factors.

So 1 has only 1 distinct factor.

All other numbers are either prime or '**composite**' (with more than 2 distinct factors).

(4) Whole numbers and Natural numbers

These are non-mathematical terms (much in the same way that a rectangle is called an oblong). It is probably best to refer instead to either $\mathbb{Z}$ (positive & negative integers, together with zero), $\mathbb{Z}^+$ (positive integers) or $\mathbb{Z}^-$ (negative integers).

Usually, Natural numbers are taken to be the positive integers, but sometimes zero is included as well (ie the non-negative integers).

In non-mathematical circles, the Whole numbers are usually taken to be the non-negative integers.

However, mathematicians tend to treat Whole numbers and Integers as being the same.

[(A) “A Mathematical Olympiad Primer” (Geoff Smith) [page 10]: the integers ($\mathbb{Z}$) are described as “the whole numbers (positive, negative and zero)”.

(B) “Numbers & Proofs” (RBJT Allenby)[page 2]: “The positive and negative whole numbers together with zero are usually referred to as the integers.” [There is ambiguity here though as to whether zero counts as a whole number!]

(5) If $cosha = b$, then $a = \pm arcoshb$ (rather than $arcoshb$).

[In order for $y = arcoshx$ to be a function (with only one possible y value for each x value), the domain of $y = coshx$ is limited to $x \geq 0$, before deriving the inverse function. Then

$y = arcoshx$ is non-negative.]

By contrast, if $sinha = b$, then $a = arsinhb$, as there is no horizontal overlap for the $sinh$ function (ie only one value of x such that $y = sinhx$, for a given y), and therefore no problem with creating the inverse function.

(6) $\int \frac{1}{\sqrt{x^2-a^2}}\ dx = arcosh\left(\frac{x}{a}\right) or \ln\,(x + \sqrt{x^2 - a^2})$ (*)

However, beware that $arcosh\left(\frac{x}{a}\right) \neq \ln\,(x + \sqrt{x^2 - a^2})$.

In fact, $arcosh\left(\frac{x}{a}\right) = \ln\,(\frac{x}{a} + \sqrt{(\frac{x}{a})^2 - 1})$

$= \ln\,(\frac{1}{a}\left[x + \sqrt{x^2 - a^2}\right])$

$= \ln\left(x + \sqrt{x^2 - a^2}\right) - lna,$

so the two answers in (*) differ by a constant

[We could write $\int \frac{1}{\sqrt{x^2-a^2}}\ dx = arcosh\left(\frac{x}{a}\right) + c$

$or \ln\left(x + \sqrt{x^2 - a^2}\right) + c_1$]