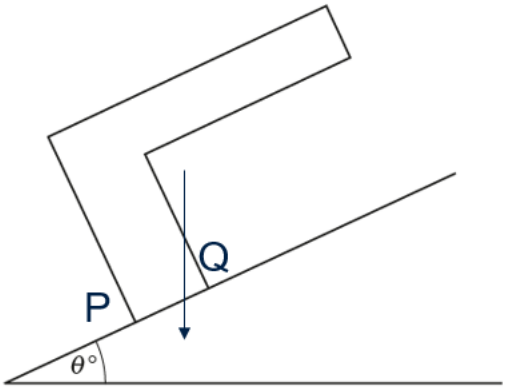
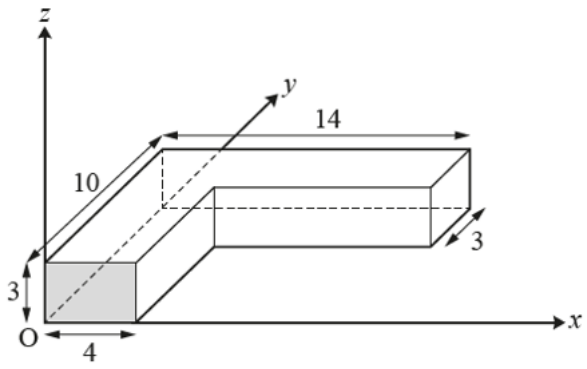
Centre of Mass – Q6 [Practice/M](18/6/23)

An object lies on a slope as shown in the left-hand diagram below. The dimensions of the object are shown in the right-hand diagram. The shaded end is in contact with the slope and the 4cm edge is along the line of greatest slope.

Given that the surface of the slope is sufficiently rough that the object will not slip, determine the minimum and maximum possible values of θ for which the object does not topple.





Solution

To find the centre of mass of the object:

Split the ornament into 2 parts: A is the lefthand cuboid, with dimensions $4 \times 10 \times 3$ (and volume 120); B is the righthand cuboid with dimensions $10 \times 3 \times 3$ (and volume 90)

The CoM of the object is:

$$\frac{1}{(120+90)}\left[120\begin{pmatrix}2\\5\\1.5\end{pmatrix}+90\begin{pmatrix}9\\8.5\\1.5\end{pmatrix}\right]=\frac{10}{210}\begin{pmatrix}105\\136.5\\31.5\end{pmatrix}=\begin{pmatrix}5\\6.5\\1.5\end{pmatrix}$$

For sufficiently large values of θ, the object may topple about P, whilst for sufficiently small values it may topple about Q.

For P, the critical point occurs when the CoM lies vertically above P; ie when $\tan\theta = \frac{5}{6.5}$, and $\theta \approx 37.569 = 37.6°$ (3sf)

For Q, the critical point occurs when the CoM lies vertically above Q; ie when $\tan\theta = \frac{(5-4)}{6.5}$, and $\theta \approx 8.7462 = 8.75°$ (3sf)

Thus the minimum and maximum possible values of θ are 8.75° and 37.6°.