Centre of Mass – Q5 [Problem/M](1/6/21)

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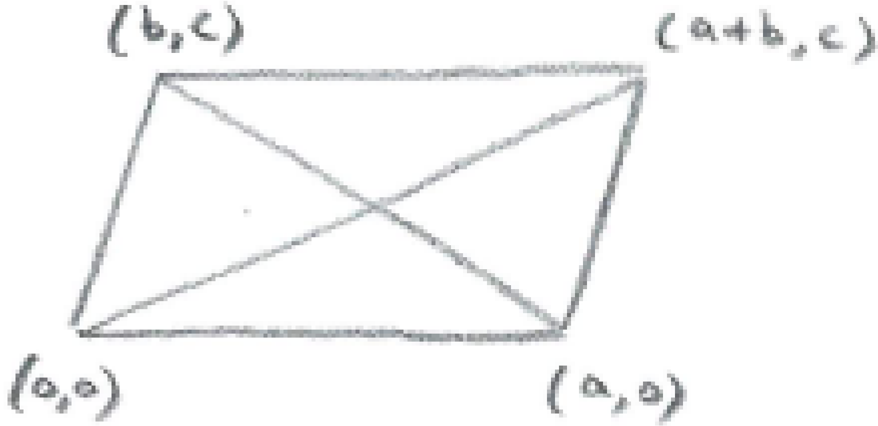
Solution



Let triangle 1 have corners

(0,0) , (a,0) & (b,c)

(and triangle 2 be the other half of the parallelogram).

$$CoM_1 = \begin{pmatrix} \frac{1}{3}(0 + a + b) \\ \frac{1}{3}(0 + 0 + c) \end{pmatrix} \ \& \ CoM_2 = \begin{pmatrix} \frac{1}{3}(a + b + [a + b]) \\ \frac{1}{3}(0 + c + c) \end{pmatrix}$$

Then centre of mass of parallelogram $= \frac{1}{2}\ (COM_1 + COM_2)$

$$= \frac{1}{6}\begin{pmatrix} 3a + 3b \\ 3c \end{pmatrix} = \frac{1}{2}\begin{pmatrix} a + b \\ c \end{pmatrix}$$

ie the mid-point of the diagonal from (0,0) to (a+b,c)