# Centre of Mass – Q3 [9 marks] (1/6/21)

## Exam Boards

OCR : Mechanics (Year 2)

MEI: Mechanics b

AQA: Mechanics (Year 2)

Edx: Mechanics 2 (Year 2)

Find the centre of mass of the region between the curve $y = x^3 - x^2$ and the $x$-axis. [9 marks]

Find the centre of mass of the region between the curve

$y = x^3 - x^2$ and the $x$-axis. [9 marks]

## Solution

$y = x^3 - x^2 = x^2(x-1)$ meets the $x$-axis at $x = 0$ & $x = 1$

[1 mark]

Note that the curve lies beneath the $x$-axis.

Total weight (signed area)

$= \int_0^1 x^3 - x^2\, dx = \left[\frac{1}{4}x^4 - \frac{1}{3}x^3\right]_0^1 = \left(\frac{1}{4} - \frac{1}{3}\right) = -\frac{1}{12}$ [2 marks]

$-\frac{1}{12}\bar{x} = \int_0^1 x(x^3 - x^2\, dx) = \left[\frac{1}{5}x^5 - \frac{1}{4}x^4\right]_0^1 = \left(\frac{1}{5} - \frac{1}{4}\right) = -\frac{1}{20}$

so that $\bar{x} = 0.6$ [3 marks]

And $-\frac{1}{12}\bar{y} = \int_0^1 \frac{y}{2}(x^3 - x^2\, dx) = \frac{1}{2}\int_0^1 (x^3 - x^2)^2 dx$

$= \frac{1}{2}\int_0^1 x^6 + x^4 - 2x^5\, dx$

$= \frac{1}{2}[\frac{1}{7}x^7 + \frac{1}{5}x^5 - \frac{2}{6}x^6]_0^1$

$= \frac{1}{2}\left(\frac{1}{7} + \frac{1}{5} - \frac{2}{6}\right)$

so that $\bar{y} = \frac{-6(30+42-70)}{210} = -\frac{2}{35} = -0.0571$ (3sf) [3 marks]

Thus the centre of mass is $(0.6, -0.0571)$