

Arithmetic Series – Q3 [Problem/M] (17/6/21)

(i) If teams A, B, C, D & E in some sporting competition have to play each other once, how many games are there in total?

(ii) Extend this to find a formula for $1 + 2 + 3 + \dots + n$

Solution

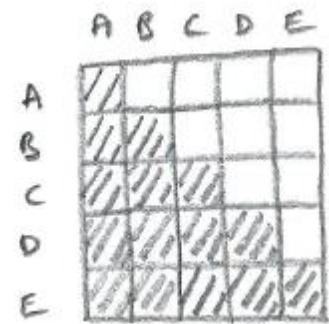
(i) AvB, AvC, AvD, AvE 4 games

BvC, BvD, BvE 3 games

CvD, CvE 2 games

DvE 1 game

Total = $1 + 2 + 3 + 4 = 10$ games



(ii) Consider the case $n = 4$

Divide the 5×5 square up into the areas X, Y & Z

Let the squares be of unit area.

Then $X = 1 + 2 + 3 + 4$,

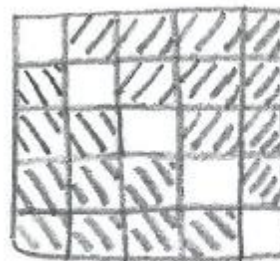
$Y = 5$ & $Z = X$




So, for $n = 4$, $X + 5 + X = 25$

Generalising this, $2X + (n + 1) = (n + 1)^2$

$\Rightarrow 2X = (n + 1)[(n + 1) - 1] = (n + 1)n$

$\Rightarrow X = \frac{n(n+1)}{2}$



total of  = X
total of  = Y
total of  = Z